

# Hybrid Control of Structures Using Fuzzy Logic

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**Abstract:** *This investigation examined the application of control algorithms based on fuzzy logic to a class of hybrid structural control systems. The investigation included both analytical and experimental verification of the fuzzy control algorithm. The objective of the hybrid system under investigation is to obtain an ideal sliding system with perfect base isolation. As the hybrid system approaches the state of ideal isolation, the effects of imperfections, signal noise, uncertainties, modeling errors, and compensation errors start to play a dominant role in the control performance. Fuzzy logic (or fuzzy set theory) provides a simple framework to capture the effects of nonlinearities and uncertainties in a real problem without an explicit model of the plant or controller.*

*The applicability of this approach was first investigated analytically and then verified using a benchmark experimental model consisting of a 1:4 scale sliding-base isolated system controlled at its base by a servohydraulic actuator with a digital computer to provide the control signal. The fuzzy controller used feedback from either the acceleration of the moving foundation or the force at the interface to produce control forces in a series of shaking-table tests. The results from this study show the feasibility of the implementation of fuzzy logic to highly nonlinear problems.*

## 1 INTRODUCTION

Much of the early research and development in active structural control was based on adapting algorithms from mechanical and electrical control systems to civil engineering problems.<sup>14</sup> The effectiveness of these linear algorithms in structural systems was demonstrated in tests on scale laboratory models and on a full-scale prototype.<sup>11,14</sup>

Hybrid structural controllers, which combine passive energy dissipative systems with active controllers, are more attractive for immediate implementation than fully active devices. An extended survey of hybrid systems<sup>13</sup> has shown that such systems are more complex than linear active controllers. Hybrid isolated structures tend to have nonlinear response, which makes accurate determination of the structural parameters and excitation characteristics an important factor.<sup>15,20,21</sup> In addition, the response measurements are contaminated with noise. The combination of modeling errors, uncertainty in the identification, and noise may reduce the effectiveness and robustness of traditional controllers, particularly when the feedback is small. Under the latter condition, the controller is essentially reacting to the noise and not to the structural feedback. A number of control schemes have been used previously to control the hybrid system. These schemes included linear control algorithms<sup>9</sup> and nonlinear controllers, which implicitly<sup>12</sup> or explicitly<sup>19</sup> accounted for noise and uncertainty.

Fuzzy controllers afford a simple and robust framework to specify nonlinear control laws that accommodate uncer-

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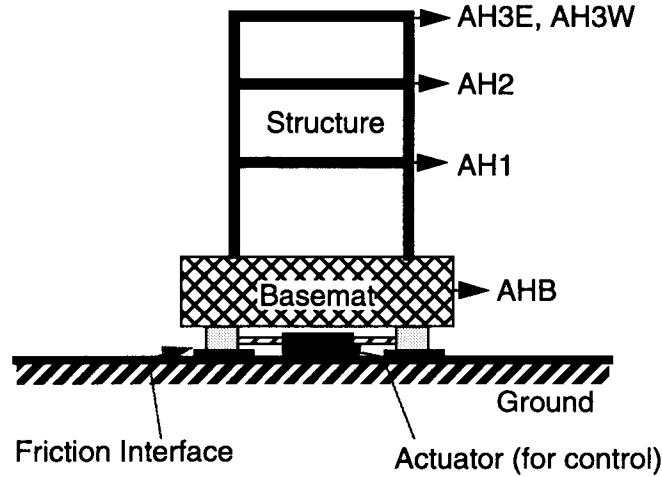


Fig. 1. Hybrid isolated structure.

tainty and imprecision. Such controllers may be implemented using a fuzzy mathematical model of the plant and the controller. In the event that linguistic descriptions of the control are available or can be formulated, fuzzy controllers may be determined without a mathematical model. Fuzzy controllers have been used widely in consumer electronics and in control applications where there are difficulties in applying classic algorithms. One of the earliest examples of control based on linguistic descriptions is the control of a steam engine.<sup>8</sup> Some other applications include automatic control of subway trains<sup>22</sup> and a fuzzy tractor-trailer backer-upper.<sup>7</sup>

Recently, there has been an increased interest in applying the concepts of fuzzy set-theory in structural control. Implementations using a linguistic synthesis approach have been proposed<sup>4,15</sup> and demonstrated to be applicable in theory<sup>16,17</sup> and in practice.<sup>15,16</sup> Another approach that has been employed, both theoretically and experimentally, is the use of a fuzzy mathematical model of the structure.<sup>5,18</sup> This study included earthquake motion prediction, structural identification, and control parameter optimization. More recently, a genetic algorithm-based approach<sup>3</sup> and a neural network approach<sup>2</sup> have been suggested for adaptive or optimal tuning of a fuzzy control system. An approach that combines a neural network and a fuzzy logic element to address actuator dynamics, time delay, and higher modes of response has been evaluated numerically.<sup>6</sup>

## 2 CONTROL OF IMPERFECT SLIDING SYSTEM

### 2.1. Problem definition

A schematic representation of a hybrid, sliding structural system is shown in Fig. 1. Such a system can be represented by the following equation:

$$\mathbf{M}(\ddot{\mathbf{x}} + \ddot{\mathbf{x}}_g \mathbf{r}) + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + f_f \mathbf{D} - u \mathbf{D} = 0 \quad (1)$$

where  $\mathbf{M}$  is the mass matrix,  $\mathbf{C}$  is the damping matrix,  $\mathbf{K}$  is the stiffness matrix,  $\mathbf{x}$  is the relative displacement,  $\ddot{\mathbf{x}}_g$  is the ground acceleration,  $\mathbf{r}$  is a vector that defines the location and direction of the ground acceleration,  $f_f$  is the friction force in the isolation,  $u$  is the control force, and  $\mathbf{D}$  is a matrix that defines the locations of the friction and control forces.

For an *ideal* hybrid control system, the control force may be designed to exactly oppose the reaction forces in the isolation interface; the control force in this case is

$$u = f_f + \mathbf{D}^T \mathbf{C} \dot{\mathbf{x}} + \mathbf{D}^T \mathbf{K} \mathbf{x} \quad (2)$$

Combining Eqs. (1) and (2) yields

$$\mathbf{M}(\ddot{\mathbf{x}} + \ddot{\mathbf{x}}_g \mathbf{r}) = 0 \quad (3)$$

Equation (3) implies that the total absolute acceleration of the system is always zero.

In practice, a number of errors and uncertainties combine to limit the control accuracy. The measurement of relative displacement and relative velocity is contaminated by sensor noise, and the noise in the velocity signals will be amplified if the velocities are measured indirectly, by taking derivatives of the displacements. Because of limitations in current system identification techniques, the damping and stiffness coefficients cannot be determined accurately. In the experimental model, the friction force could not be measured directly and had to be estimated. The delay, actuator dynamics, and limitations in the model of the system all affect the accuracy of the control force. The control force can only be obtained through an estimate of the prevalent forces at a given time. The estimated control force can be defined as

$$u = f_f^* + \mathbf{D}^T \mathbf{C}^* \dot{\mathbf{x}}^* + \mathbf{D}^T \mathbf{K}^* \mathbf{x}^* \quad (4)$$

where the \* indicates that the term is an estimate of the respective quantity. This results in an imperfect control in which the absolute acceleration does not vanish. Instead,

$$\mathbf{M}(\ddot{\mathbf{x}} + \ddot{\mathbf{x}}_g \mathbf{r}) = \mathbf{M}\boldsymbol{\epsilon}_t \quad (5)$$

where  $\boldsymbol{\epsilon}_t$  is the deviation from zero of the absolute acceleration and is a measure of the error in the control force. Mathematical modeling of this error is difficult using conventional approaches and may be impossible in some cases. Hence compensation for this error is complicated. A scheme that can capture and compensate for imprecision and uncertainty can be developed using fuzzy logic, which provides a simple framework to handle these errors.

## 2.2 Suggested solution

Two approaches for determining the control force have been identified: the *total force formulation* and the *incremental force formulation*. Using a Coulomb model of the friction force,

$$f_f = \mu W \operatorname{sgn} \dot{\mathbf{x}} \quad (6)$$

where  $\mu$  is the coefficient of friction and  $W$  is the total weight of the structure. Neglecting the stiffness and damping at the interface for simplicity, the control force required to achieve perfect isolation is

$$\mathbf{u} = f_f + \mathbf{D}^T \mathbf{M} \boldsymbol{\epsilon}_t \quad (7)$$

Since only an estimate of the friction force can be obtained, Eq. (7) becomes

$$\mathbf{u} = \mu^* W^* \operatorname{sgn} \mathbf{D}^T \dot{\mathbf{x}}^* + \Delta W \Delta \mu \operatorname{sgn} \mathbf{D}^T \Delta \dot{\mathbf{x}} + \mathbf{D}^T \mathbf{M} \boldsymbol{\epsilon}_t \quad (8)$$

where  $\Delta$  is used to signify the difference between the estimate and the actual value of the corresponding quantity. Equation (8) may be rewritten as

$$\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2 \quad (9)$$

where

$$\mathbf{u}_1 = \mu^* W^* \operatorname{sgn} \mathbf{D}^T \dot{\mathbf{x}}^* \quad (10)$$

$$\mathbf{u}_2 = \Delta \mu \Delta W \operatorname{sgn} \mathbf{D}^T \Delta \dot{\mathbf{x}} + \mathbf{D}^T \mathbf{M} \boldsymbol{\epsilon}_t \quad (11)$$

Here  $\mathbf{u}_1$  is the component of the control force that opposes the estimated friction force, and  $\mathbf{u}_2$  is the component of the control force that compensates for the errors in the estimation.

In the *total force formulation*, the control algorithm determines the entire value of the control force by approximating the friction force and measuring the remnant absolute acceleration. The estimate of the friction force at large velocities is usually quite good, so  $\mathbf{u}$  may be estimated with reasonable accuracy.

At low velocities, the signum term in  $\mathbf{u}_1$  may introduce a sufficient error, rendering the control completely ineffective. A direct measurement of the difference in the force,  $\Delta \mu \Delta W \operatorname{sgn} \mathbf{D}^T \Delta \dot{\mathbf{x}}$ , and of the absolute acceleration is sufficient to determine the change in the control force from one step to another. This is defined as the *incremental force formulation*. The fuzzy logic strategy is applied to the total form, since it can better quantify the uncertainty in the direction of the friction force and the absolute acceleration.

## 3 FUZZY CONTROL

The fuzzy controller uses a form of quantification of imprecise information (input fuzzy sets) to generate by an inference scheme, which is based on a knowledge base of control, a precise control force to be applied on the system. The logical controller is made of three main components, as shown in Fig. 2: (1) a fuzzification component, where the information is quantified by means of fuzzy sets, (2) a fuzzy logic processing component, which converts the input fuzzy sets into control force fuzzy sets, through rules collected in the knowledge base, and aggregates the resulting fuzzy sets, and (3) a defuzzification component, where the output fuzzy information (aggregated fuzzy set) is converted into a precise value.

Fuzzy sets are groups of objects where each member of the set has an associated *degree of membership*  $\mu$  in that set, as shown in Fig. 3. A fuzzy set may have ill-defined boundaries. The domain of definition of the fuzzy sets is known as the *universe of discourse (UofD)*; for example, in the case of numbers, the UofD could be the axis of real numbers. A set with a definite boundary, where elements inside the boundary have complete membership in the set (membership values of 1.0) and those outside the boundary have no membership, is defined as a *crisp set*. The universe of discourse can be covered by a number of fuzzy sets with overlapping boundaries, and a number on the UofDs can be a member of one or more fuzzy sets, with a different membership value in each of these sets.

The advantage of this quantification is that the fuzzy sets can be represented by a unique linguistic expression, such as small, large, etc. The linguistic representation of a fuzzy set is known as a *term*, and a collection of such terms defines a *termset*, or library of fuzzy sets. The linguistic representation simplifies the specification of the control laws through the use of linguistic rules. The method of processing these rules to determine a control action is known as *linguistic synthesis*.<sup>8,23</sup> A fuzzy controller that implements linguistic synthesis is made up of two components: (1) a knowledge base of linguistic rules, or *rulebase*, which relates a measured datum to a control force, and (2) an *inference scheme*, or mechanism to process the rules. For a system of multiple controller inputs, with each input corre-

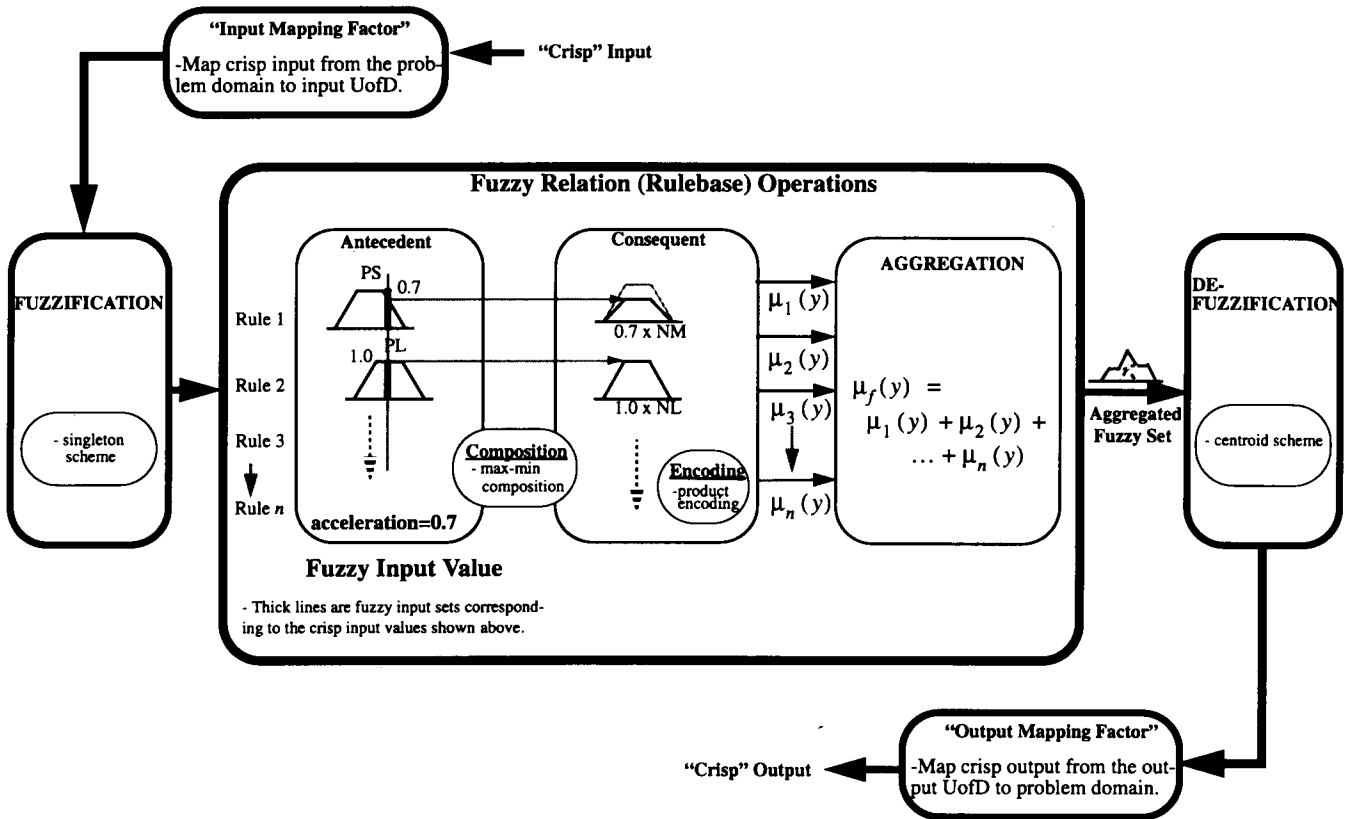


Fig. 2. Schematic of the fuzzy control algorithm.

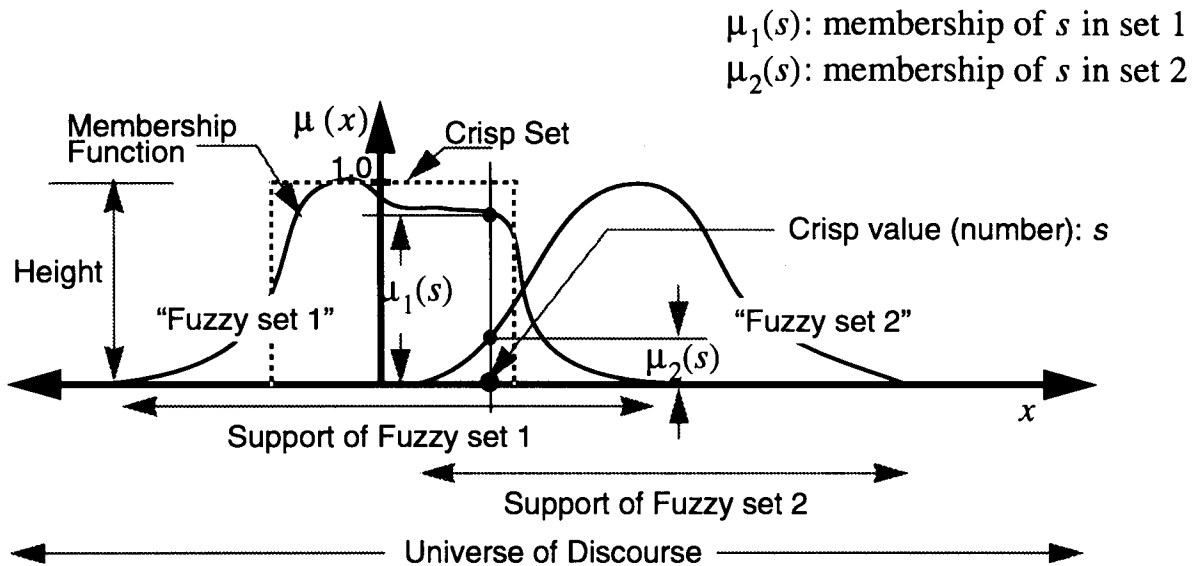


Fig. 3. Fuzzy sets compared with a crisp set.

sponding to one fuzzy variable, which is defined over its own UofD, a fuzzy rule  $R_i$  in the rulebase that relates the input  $V_i$  to an output  $u$  will have the form

$$R_i: \text{IF } v_1 \text{ is } S_{1,j_1} \text{ AND } \dots v_k \text{ is } S_{k,j_k} \dots \\ \text{AND } v_n \text{ is } S_{n,j_n} \text{ THEN } u \text{ is } U_m \quad (12)$$

where  $v_k$  is the  $k$ th input fuzzy variable,  $S_{k,j_k}$  is the  $j_k$ th fuzzy set defined on the  $k$ th UofD,  $u$  the output fuzzy variable, and  $U_m$  is the  $m$ th fuzzy set on the output UofD. A fuzzy rule represents a fuzzy relation, or a fuzzy implication known as a *fuzzy mapping*. Assuming that the fuzzy mapping is generated using a product-encoding scheme, then the fuzzy mapping between the input  $\hat{v} = (v_1, \dots, v_n)$  and the output  $u$  is defined as

$$\mu_{R_i}(\hat{v}, u) = \mu_{S_{1,j_1}}(v_1) \wedge \dots \wedge \dots \wedge \mu_{S_{n,j_n}}(v_n) \mu_{U_m}(u) \quad (13)$$

where  $\wedge$  is the fuzzy AND operator. Given an arbitrary input set  $A'$ , an output fuzzy set for control  $U'_i$  can be inferred from the  $i$ th rule by a *composition* scheme.<sup>23</sup> In this study, *max-product* composition was used. This is defined as

$$\mu_{U'_i}(u) = \max_{\hat{v} \in C} [\mu_{A'}(\hat{v}) \mu_{R_i}(\hat{v}, u)] \quad (14)$$

Each rule yields one output fuzzy set  $U'_i$ ; therefore, the resultant fuzzy output set  $U'$  represents the combined effect of all the rules in the rulebase. This resultant set is obtained by combining the individual output sets with an appropriate *aggregation* scheme. In the aggregation scheme employed, the membership function  $U'_i$  is defined as

$$\mu_{U'}(u) = \frac{[\mu_{U'_1}(u) + \dots + \mu_{U'_M}(u)]}{\max_{u \in D} [\mu_{U'_1}(u) + \dots + \mu_{U'_M}(u)]} \quad (15)$$

where  $D$  is the output universe of discourse. The control force is determined by a process of *defuzzification* of the final output set  $U'$ . In this study, the defuzzified output was obtained by determining the centroid of the membership distribution of  $U'$ . A single-input, single-output (SISO) fuzzy controller using singleton fuzzification, max-product composition, product encoding, sum aggregation, and centroid defuzzification is given as

$$t = \frac{\int_{(y \in 0)} y \left\{ \sum_{i=1}^R [\mu_{A_i}(s) \mu_{B_i}(y)] \right\} dy}{\int_{(y \in 0)} \sum_{i=1}^R [\mu_{A_i}(s) \mu_{B_i}(y)] dy} \quad (16)$$

where  $\mu_{A_i}$  and  $\mu_{B_i}$  are the input and output membership functions in the  $A_i$  and  $B_i$  reference sets from the  $i$ th rule of the rulebase,  $R$  is the number of rules in the rulebase,  $s$  is the mapped image of a crisp input to the fuzzy controller,  $y$  is an element on the output UofD, and  $t$  is the crisp output from the controller. This relationship enables extremely fast

processing of the fuzzy control, as required for real-time operations. A flowchart showing the real-time implementation of the controller is shown in Fig. 4.

Fuzzy processing enables the quantification of irregular input by sets that can be treated according to their significance. For example, a fuzzy set representing numbers near zero may contain highly unreliable information. A rule in the rulebase can be defined such that the control force will be zero without having a detrimental effect on the system. Similarly, any set can be treated differently from any other set. The fuzzy controller has several advantages:<sup>16</sup> (1) it can discriminate between groups of input values of high or low importance and treat them accordingly with a small number of operations, (2) it can easily eliminate actions that may be detrimental to the system, such as extreme control forces or uncertain input near zero, (3) using fuzzy sets of trapezoidal shape, input signals are implicitly filtered, and (4) the output aggregation and defuzzification schemes reduce the influence of input noise and computer imprecision.

#### 4 DEFINITION OF CONTROL PARAMETERS

The study focused on the physical implementation of an SISO fuzzy controller. Several parameters were determined to be important for developing and tuning the controllers. The particular control parameters studied were as follows:<sup>16</sup>

1. *Mapping factors*: Mapping factors are scale factors that multiply a normalized UofD, as shown in Fig. 5a. This facilitates the increase or decrease of the base of each of the reference fuzzy sets used to encode the rules. The *input* and *output mapping factors* function as linear control gains, while the fuzzy system acts as a nonlinear modifier of the overall gain of the controller.<sup>16</sup> These factors affect the properties of the controller and play an important role in the dynamics of the fuzzy controller.<sup>10</sup>
2. *Fuzzy-sets selection*: The type, number, and distribution of the fuzzy sets determine the input-output relationship of the controller through corresponding rules. A suitable selection of the types of sets, the numbers of sets, and the distribution of the sets along the real number axis produces either a linear or a nonlinear input-output relationship, as shown in Figs. 5b, 6, and 7.

Analytical studies of the importance of these parameters were performed to determine the feasibility of applying such controllers.<sup>16</sup> The influence of mapping factors was found to be significant in determining a range of feasible factors for the control of ground motions. For one ground motion, an acceleration performance index, based on the peak response, was defined and plotted, as shown in Fig. 8. The surface that was obtained was compared with similar

**Legend**

- $x$ : measured input (crisp value)
- $N$ : numerator in Equation (18)
- $M$ : denominator in Equation (18)
- $R$ : total number of rules in database
- $F_{inp}, F_{out}$ : input and output mapping factors
- $\mu_{A_i}(s)$ : membership of  $s$  in the  $i$ th input (reference) set
- $G_i$ : area under membership function of  $i$ th output (reference) set
- $q_i$ : centroid of  $i$ th output (reference) set
- $C_{force}$ : the (crisp) control output

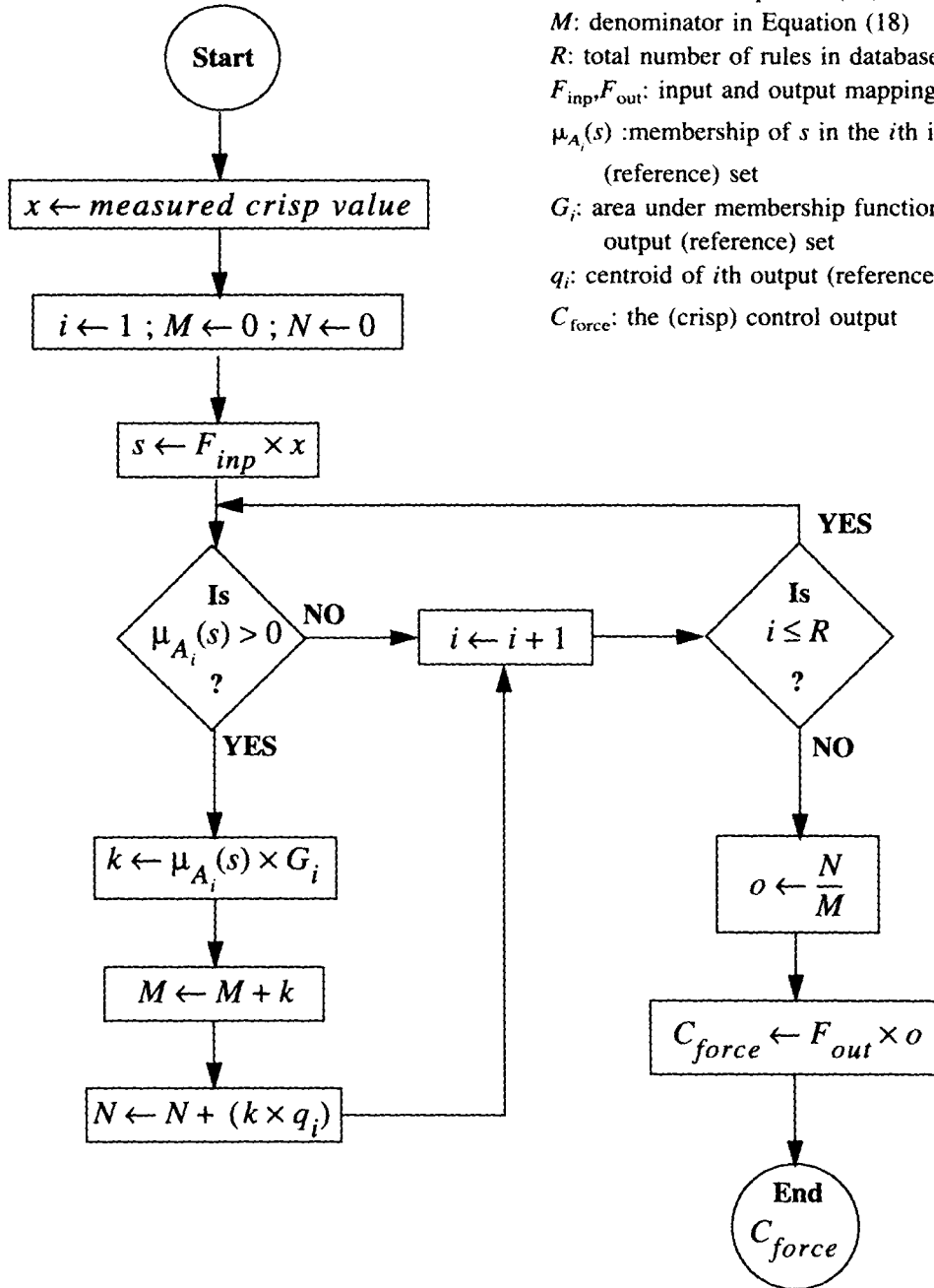


Fig. 4. Flowchart for implementation of a SISO fuzzy controller.

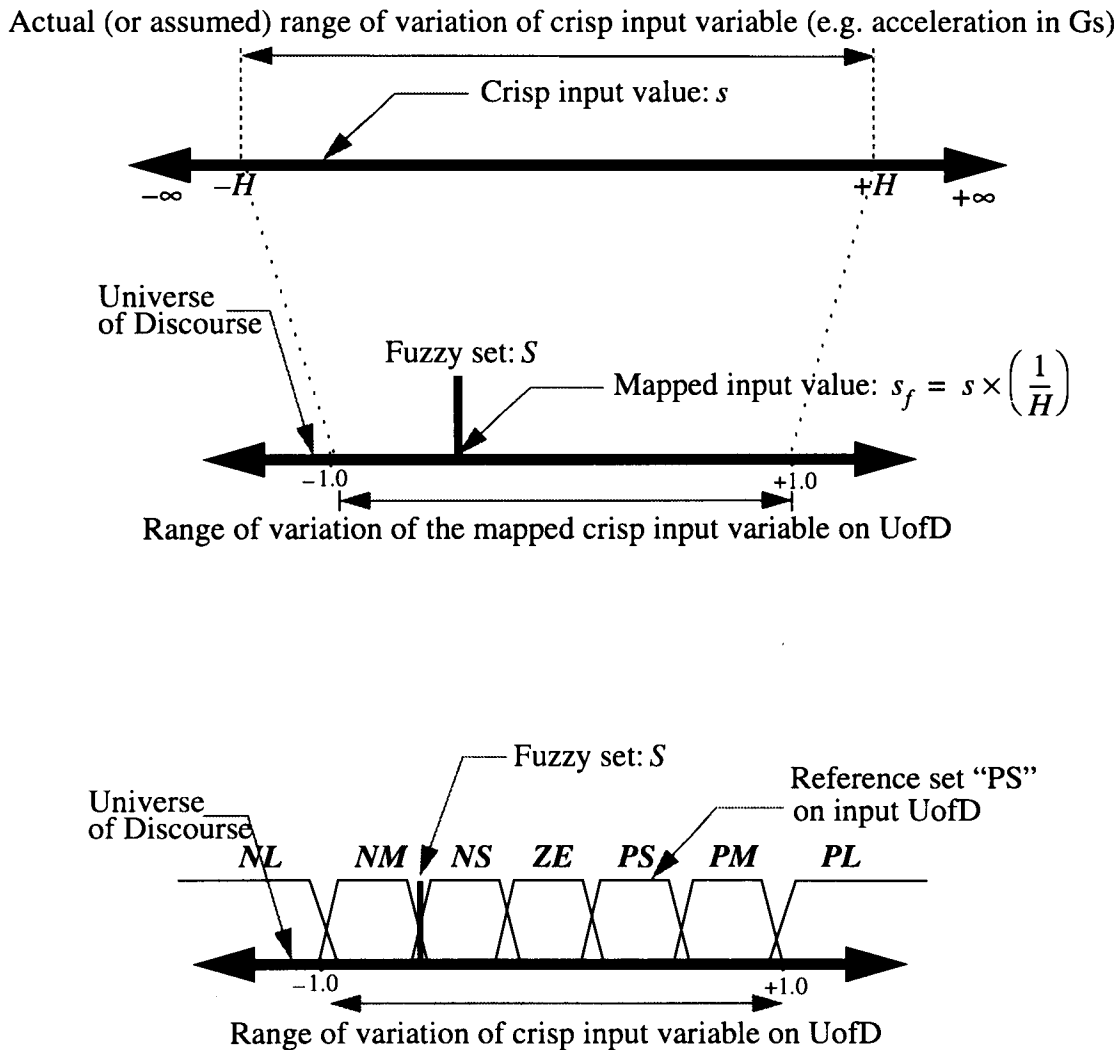


Fig. 5. Input processing. (a) Input mapping and fuzzification. (b) Degree of compatibility: input fuzzy set and reference sets.

surfaces for other earthquakes. In all cases, a common flat region of small response existed, which, in turn, defined the feasible ranges for the mapping factors. A parametric study of the influence of the selection of fuzzy sets showed that linear and nonlinear control laws can be generated by changing the position and the size of the base of the fuzzy sets.<sup>16</sup>

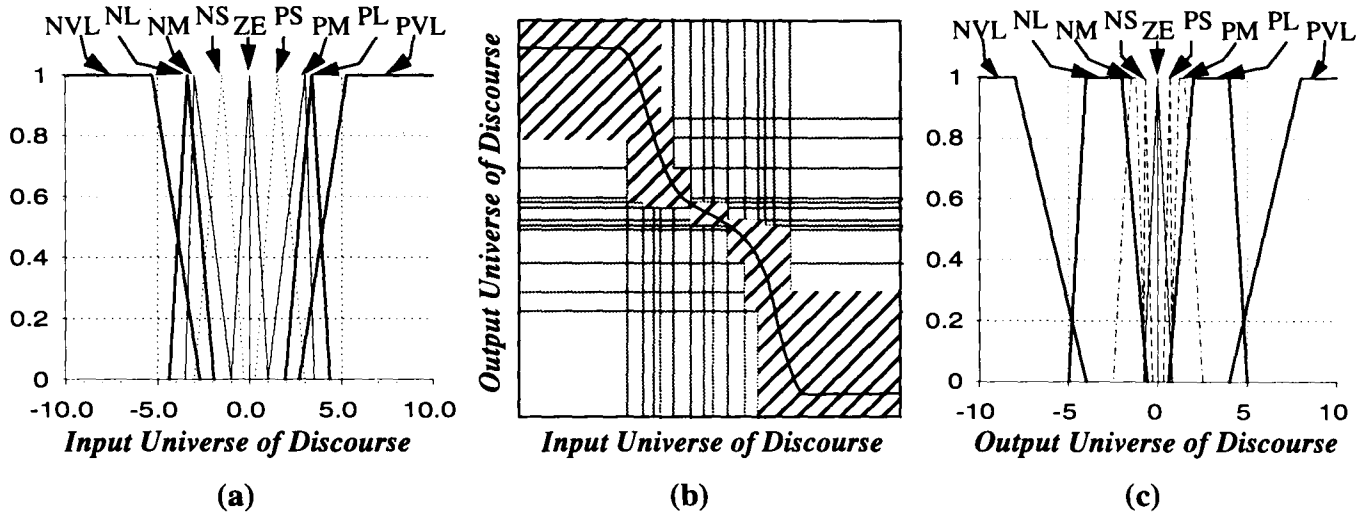
## 5 EXPERIMENTAL STUDY

### 5.1 System description

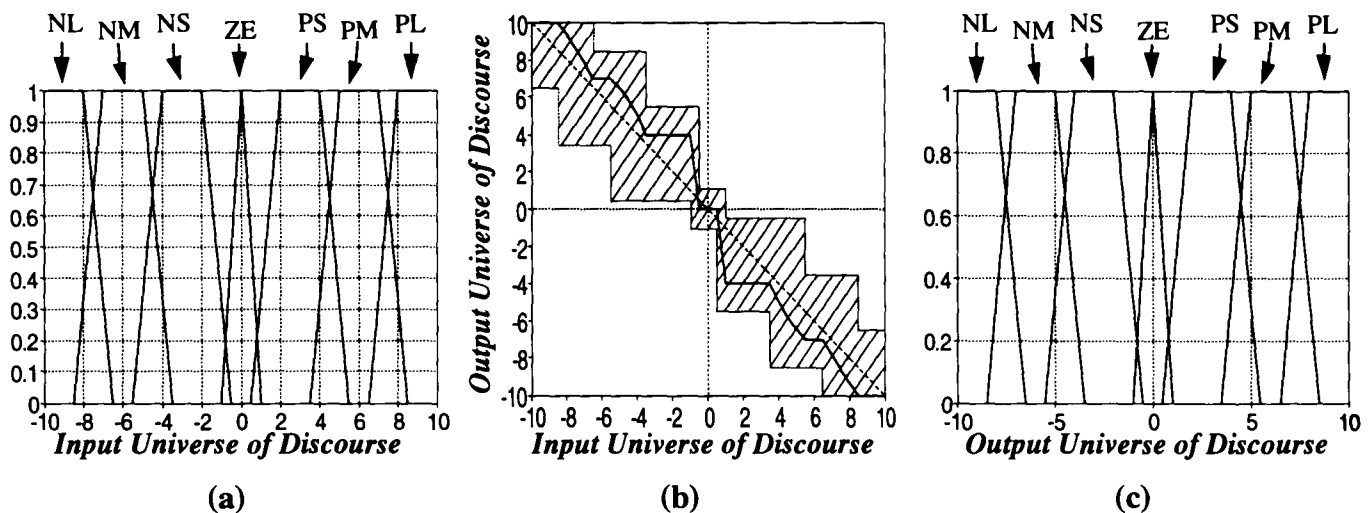
The fuzzy controller developed above was used to control a 1:4 scale-model structure. Three structural configurations

were tested, as shown in Fig. 9: (a) a sliding basemat, (b) a sliding basemat supporting a single-degree-of-freedom (SDOF) flexible structure, and (c) a three-degree-of-freedom (3-DOF) structure on the sliding basemat. Case (a) represents a bridge deck on stiff piers, while cases (b) and (c) represent a base-isolated flexible structure. The SDOF and 3-DOF flexible structures used the same model, and the SDOF structure was produced by rigidly bracing the upper stories of the structure. The properties of the structure are listed in Table 1.

The basemat was mounted on four Teflon–stainless steel isolation pads. Shear springs, located on either side of the base, provided restoring forces in the isolation system. The experimentally determined properties of the sliding interface and the shear springs are shown in Table 2.



**Fig. 6.** Fuzzy set distribution for the smooth, nonlinear profile. (a) Reference fuzzy sets on input (acceleration) universe of discourse. (b) Control relations and effective curve. (c) Reference fuzzy sets on output (force) universe of discourse.



**Fig. 7.** Fuzzy set distribution for the piecewise linear profile. (a) Reference fuzzy sets on input (acceleration) universe of discourse. (b) Control relations and effective curve. (c) Reference fuzzy sets on output (force) universe of discourse.

To provide control forces, a servohydraulic actuator was positioned between the basemat and the ground. The actuator was connected to the basemat through a sliding interface that allowed for vertical and lateral motions due to small misalignments. The actuator's servovalves were controlled by an analog servocontroller, which was controlled, in turn, by the fuzzy controller.

The instrumentation for the experimental tests included accelerometers, displacement transducers, and load cells. The accelerometers were used to measure the absolute acceleration of the basemat and each floor. The displacement transducers were used to measure the absolute displacements of the floors and the basemat and the relative displacement of the basemat. The load cells, which were



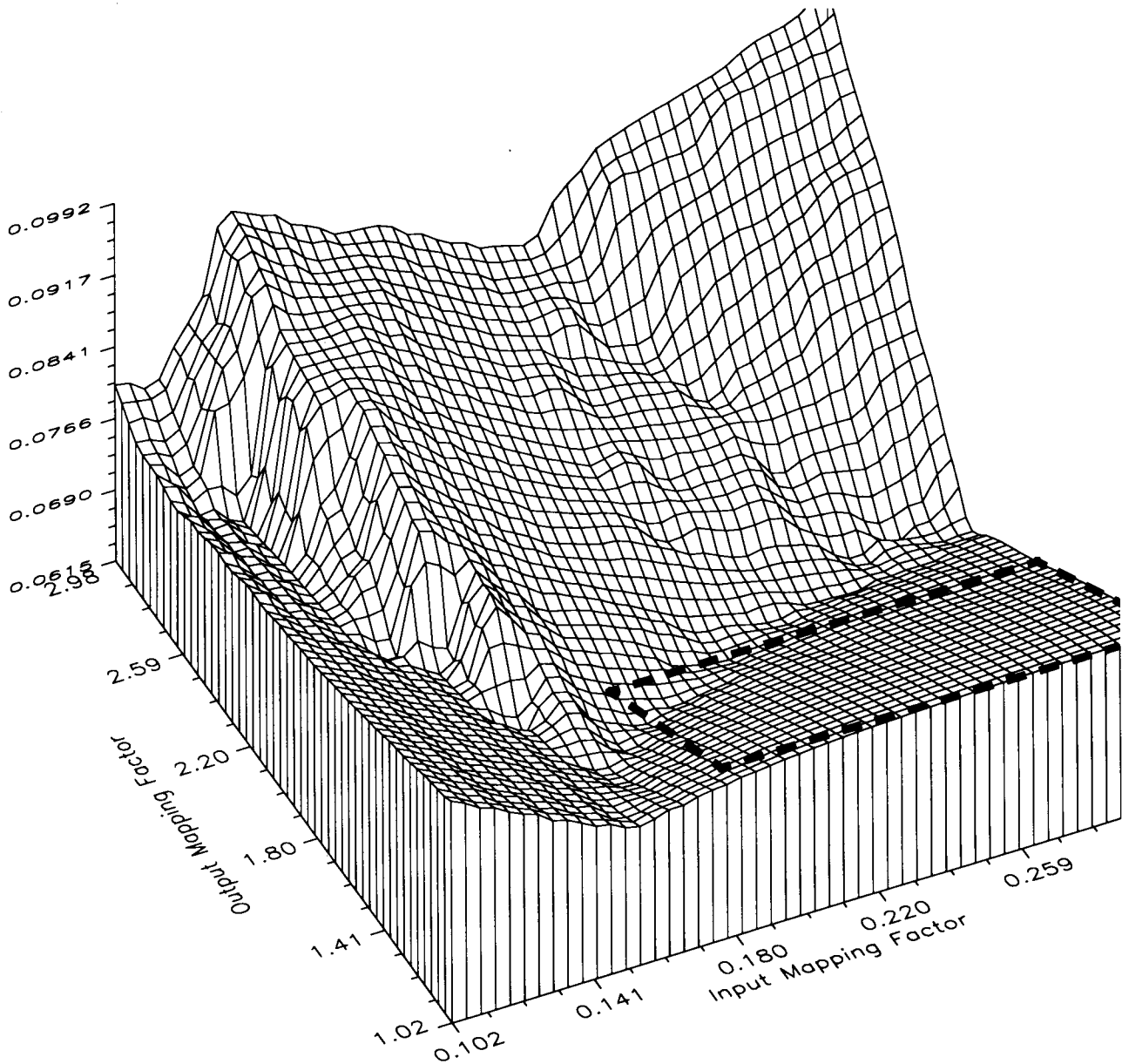


Fig. 8. Peak responses from varying input and output mapping factors (feasible control range indicated by dashed frame).

placed at either end of the actuator rod, were used to measure the force applied by the actuator. The response data were recorded using a 100-Hz sampling rate.

The fuzzy control algorithm was implemented on an Intel 80386-based computer running at 25 MHz. Analog-to-digital and digital-to-analog signal conversion was provided by an 8-bit data-acquisition board in the computer. The fuzzy control algorithm and the data-acquisition board driver modules were programmed using C.

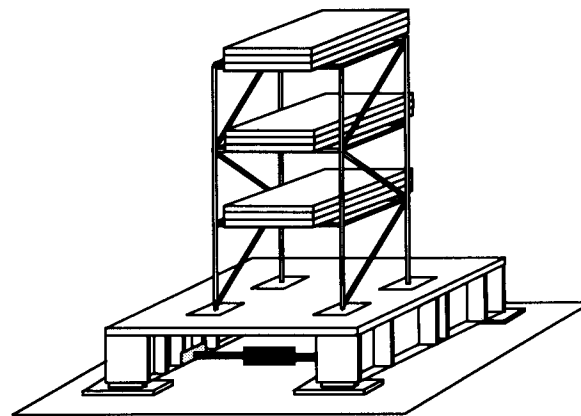
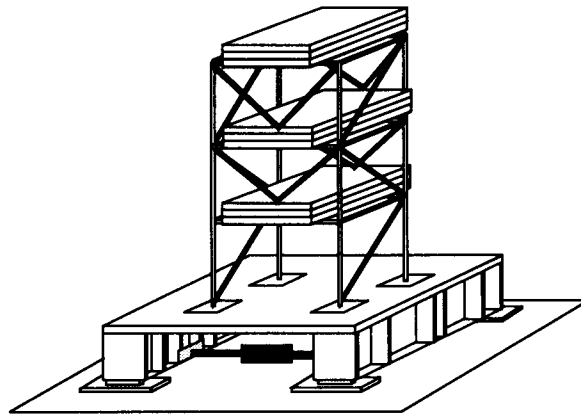
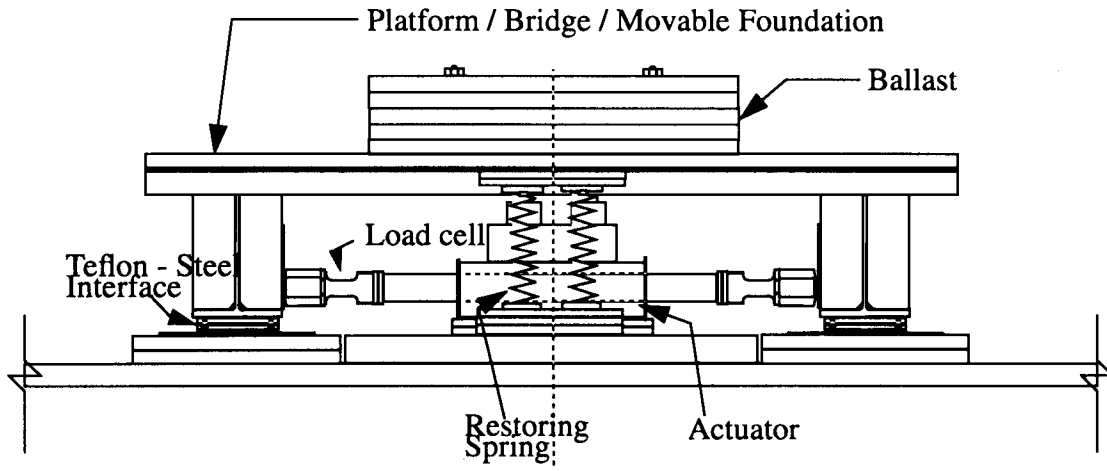
## 5.2 Analytical model

The system was modeled using a simplified lumped-mass model. The equation of motion is

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + f_f\mathbf{D} = u\mathbf{D} - \mathbf{M}\ddot{\mathbf{x}}_g\mathbf{r} \quad (17)$$

The friction force at the isolation interface is now given as

$$f_f = \mu_s WZ \quad (18)$$



**Fig. 9.** Model structure configurations. (a) Sliding rigid basemat. (b) Flexible structure with top two stories braced (SDOF). (c) Flexible structure with all stories unbraced (3-DOF).

**Table 1**  
Structure properties

Model parameters (1)	Rigid System (2)	Basemat and 1-DOF System (3)	Basemat and 3-DOF System (4)
Basemat weight (kN)	10.4	10.4	11.3
Total superstructure weight (kN)	27.4*	28.3†	28.3†
Total weight (kN)	37.8	38.7	39.6
Frequency (Hz)			
Rigid mode	0.69		
1st mode		3.20	1.95
2nd mode			6.70
3rd mode			12.30

\*Weight of the ballast used to provide the required mass.

†Based on three floors weighing 9.4 kN each.

**Table 2**  
Identification of passive and hybrid interface parameters

Friction model parameters	
Coefficient of friction at low velocities, $\mu_{\min}$	0.03
Coefficient of friction at high velocities, $\mu_{\max}$	0.06
Coefficient of variation with velocity, $\mu_a$ (s/mm)	0.09
Total effective bearing area (mm <sup>2</sup> )	126.66
Average bearing pressure (MPa)	82
Spring characteristics	
Free length (mm)	190.5
Average total lateral stiffness (kN/mm)	0.142
Initial compression (mm)	6.35
Actuator	
Maximum force capacity (kN)	24.5
Maximum physical stroke (mm)	76.2
Maximum permitted stroke (mm)	63.5
Servo valve peak flow rate (liters/min)	113.7
Peak velocity attainable (m/s)	1.27
Load cell	
Calibrated range (kN)	±44.5
Initial prestress (kN)	26.7

where  $\mu_s$  is the coefficient of sliding friction,  $W$  is the weight of the structure, and  $Z$  is a hysteretic parameter that models the stick-slip behavior at the sliding interface. The coefficient of friction can be modeled<sup>1</sup> as

$$\mu_s = \mu_{\max} - (\mu_{\max} - \mu_{\min})e^{-\mu_a|x|} \quad (19)$$

where  $\dot{x}$  is the relative velocity at the interface,  $\mu_{\max}$  and  $\mu_{\min}$  are the maximum and minimum coefficients of friction, and  $\mu_a$  characterizes the variation of the friction with respect to velocity. The hysteretic parameter is modeled<sup>1</sup> as

$$\dot{Z} = \frac{\dot{x}\{\alpha - |Z|\eta[\gamma \operatorname{sgn}(\dot{x}Z) + \beta]\}}{Y} \quad (20)$$

where  $\alpha = 1.0$ ,  $\gamma + \beta = 1.0$ ,  $\eta = 2.0$ , and  $Y$  is the yield displacement of the sliding surface.

### 5.3 Fuzzy controller

A static fuzzy controller with time-invariant parameters was evaluated experimentally. The fuzzy controller was based on the singleton fuzzification scheme, with max-min composition, product encoding, sum aggregation, and centroid defuzzification, as described earlier. Two variations of the control law were developed. One variation had a piecewise linear relationship between the input and output, while the second had a smooth nonlinear relationship.

The control command for the fuzzy controller at time  $t$  is

$$U(t) = f_1(\epsilon_b(t - \Delta t)) \quad (21)$$

where  $\epsilon_b(t - \Delta t)$  is the absolute acceleration at a time instant  $\Delta t$  prior to the application of the control force. Function  $f_1$  is based on the total force formulation, described by Eq. (9).

The response of the system is highly nonlinear because of the friction interface and the response of the actuator. The fuzzy controller generates a nonlinear control signal, which is shown graphically in Fig. 6b. The shaded region represents the fuzzy mapping performed by the rules, given the input and output universes of discourse shown. The shaded region may be regarded as a *fuzzy graph* of the fuzzy mapping produced by the controller. The solid line is the crisp

function represented by the controller and is the actual control law.

The rulebase is shown in Table 3, and the linguistic terms used to encode these rules are shown in Table 4. The distribution of the corresponding reference sets on the input and output universes of discourse are shown in Fig. 6a and c for the smooth nonlinear profile and in Fig. 7a and c for the piecewise linear profile.

## 6 EXPERIMENTAL AND ANALYTICAL RESULTS

The structural models were tested using simulated earthquakes and white noise base motion generated by a seismic simulator at SUNY at Buffalo. The earthquake records used to verify the control feasibility included El Centro (1940), Taft (1952), Pacoima Dam (1971), and Hachinohe (1968).

In the experimental study, the controller that was used for the rigid basemat also was used to control the multi-degree-of-freedom structure by adjusting the mapping factors appropriately; the controller was not adjusted to include information from the superstructure or any observer computations. The results for the Pacoima Dam earthquake motion, scaled to 0.36g, are shown in Figs. 10, 11, and 12. A comparison of an analytical simulation with the experimental performance for the rigid basemat configuration is shown in Fig. 13. These results can be extrapolated to a full-scale prototype based on suitable similitude considerations.

The controller was tuned by determining the mapping factors that yielded the best absolute acceleration response. All other control parameters were maintained unchanged. The total time delay in the feedback loop was 27 ms. This time delay was made up of the following components: an analog filter delay of 20 ms, a maximum computation delay of 1.5 ms, and an actuator response delay of 5.5 ms. The analog filter was needed to eliminate digital aliasing and to

**Table 3**  
Linguistic representation of fuzzy rulebase

<i>Rule No.</i> (1)	<i>Linguistic Rule</i> (2)
1	IF <i>acceleration</i> is <b>ZE</b> THEN <i>force</i> is <b>ZE</b>
2	IF <i>acceleration</i> is <b>NVL</b> THEN <i>force</i> is <b>PVL</b>
3	IF <i>acceleration</i> is <b>NL</b> THEN <i>force</i> is <b>PL</b>
4	IF <i>acceleration</i> is <b>NM</b> THEN <i>force</i> is <b>PM</b>
5	IF <i>acceleration</i> is <b>NS</b> THEN <i>force</i> is <b>PS</b>
6	IF <i>acceleration</i> is <b>PS</b> THEN <i>force</i> is <b>NS</b>
7	IF <i>acceleration</i> is <b>PM</b> THEN <i>force</i> is <b>NM</b>
8	IF <i>acceleration</i> is <b>PL</b> THEN <i>force</i> is <b>NL</b>
9	IF <i>acceleration</i> is <b>PVL</b> THEN <i>force</i> is <b>NVL</b>

**Table 4**  
Linguistic terms for control

<i>Linguistic Term</i> (1)	<i>Abbrev.</i> (2)
Negative very large	NVL
Negative large	NL
Negative medium	NM
Negative small	NS
Zero	ZE
Positive small	PS
Positive medium	PM
Positive large	PL
Positive very large	PVL

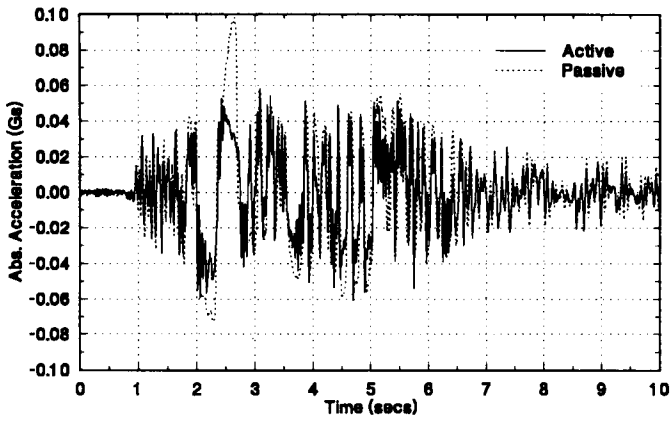
remove high-frequency noise and harmonics generated by the shaking system. The time-delay compensation was implicitly handled, as a consequence of tuning the fuzzy controller under such conditions.

Comparisons of the performance of the hybrid system with the performance of the passive system for various ground motions are shown in Table 5. The reductions in the peak absolute acceleration of the hybrid system are 20% to 40% greater than for the passive system alone when using a moderate control force. Though control of the relative displacement response of the hybrid system was not an explicit objective, it was similar to the response of the passive system.

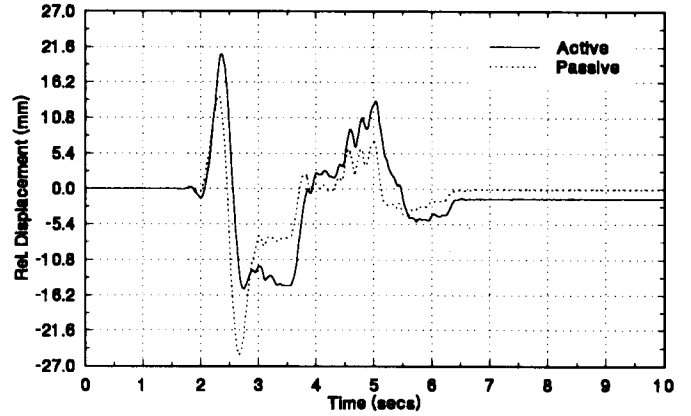
Comparisons of the peak analytical and experimental responses are shown in Table 6 and in Fig. 13. The products of the mapping factors were larger in the experiment than in the analytical study. This was probably due to having to overcome the actuator dynamics, which were not included in the analysis. The input mapping factors tended to be lower in the experimental study than in the analytical one. This was necessary to reduce the sensitivity of the controller to noise.

For the case of the rigid basemat, shown in Fig. 10, the control effectively reduced the peak absolute acceleration. The steady-state error in the relative displacement was higher for the hybrid case than for the passive case, but the deviation from zero was small. This may be explained by the lack of sensitivity to changes around the setpoint, because the input mapping factor had to be chosen small enough to eliminate a controller instability, which was due to noise in the feedback loop when the signal was small. This resulted in an increased influence of the ZE set, and no control action was taken when the input was in this set. The high frequencies superimposed on the response are due primarily to the effect of delay.

The flexible SDOF structure has an acceleration response that is out of phase with the basemat when its response is at

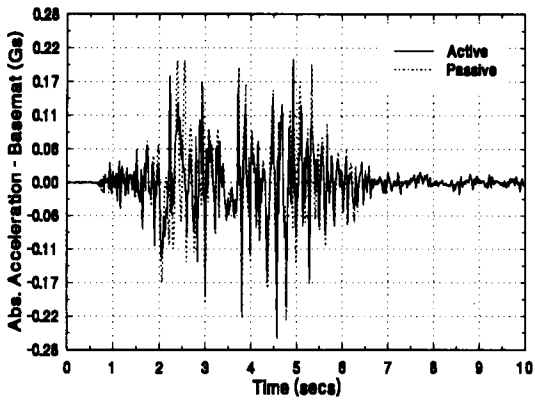


(a)

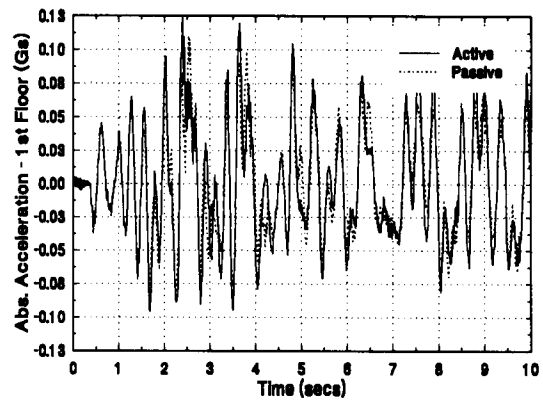


(b)

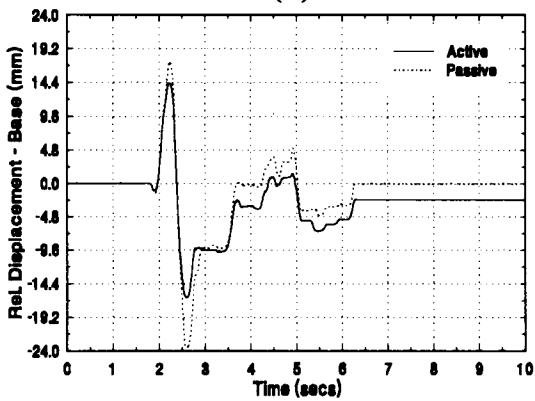
Fig. 10. Experimental results for rigid structure configuration, Pacoima Dam ground motion (PGA = 0.36g).



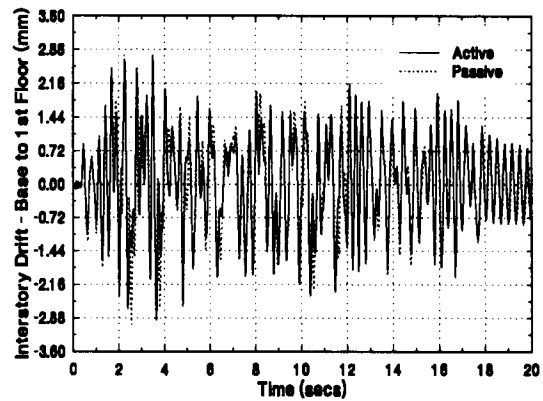
(a)



(b)



(c)



(d)

Fig. 11. Experimental results for SDOF structure configuration, Pacoima Dam ground motion (PGA = 0.36g).

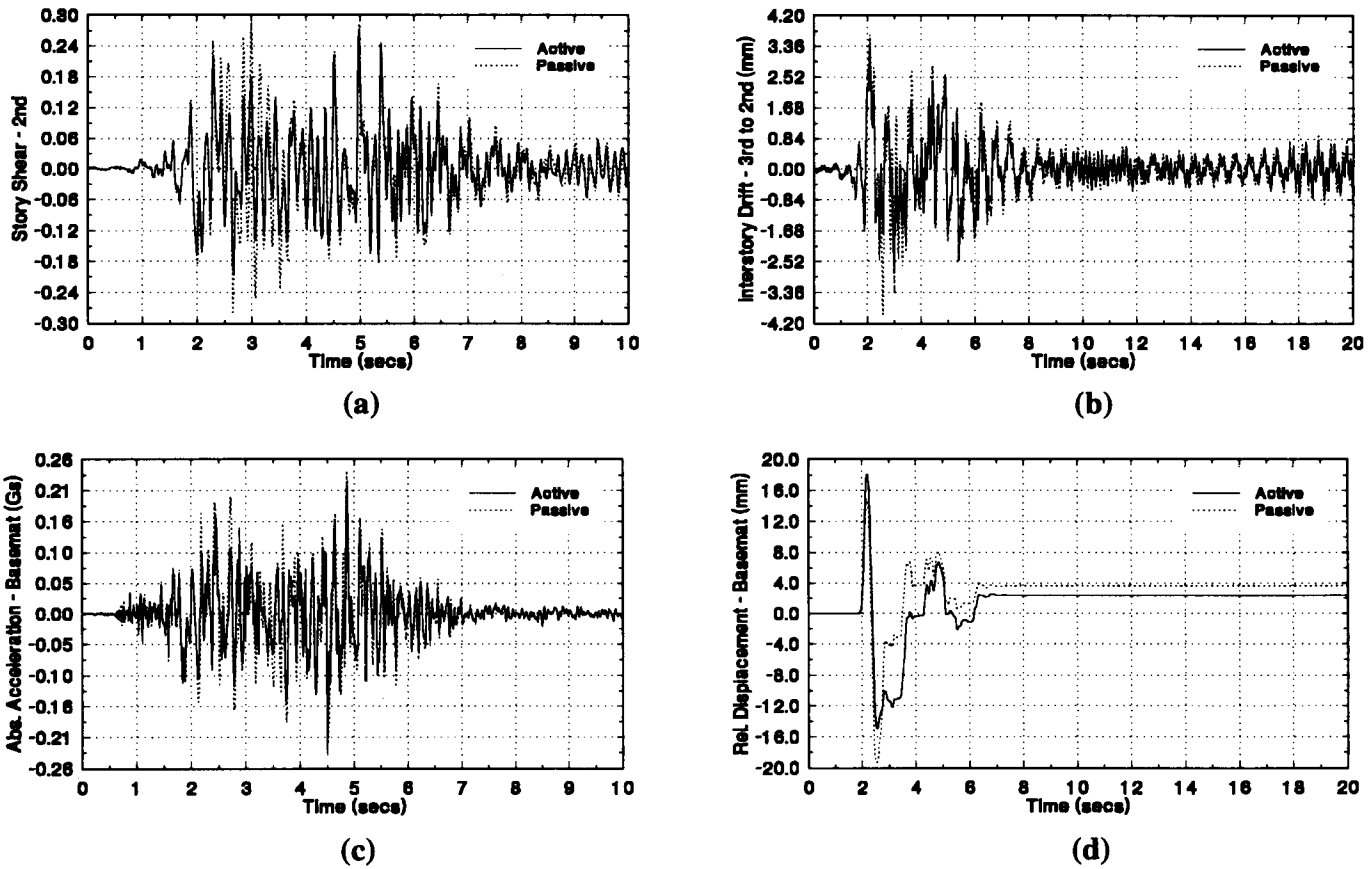


Fig. 12. Experimental results for 3-DOF structure configuration, Pacoima Dam ground motion (PGA = 0.36g).

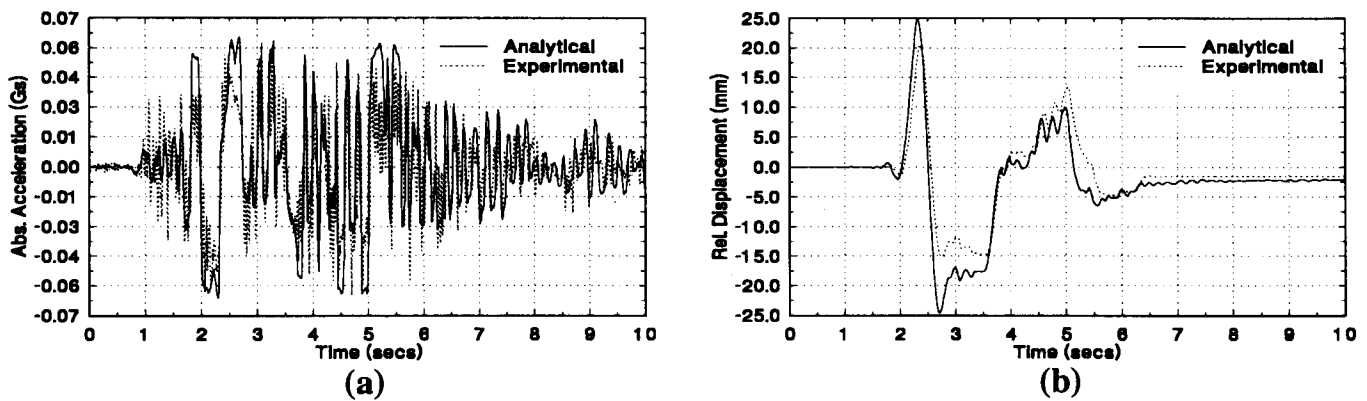


Fig. 13. Comparison of analytical and experimental results for the rigid structure configuration.

the natural frequency of the structure. Since the structure is expected to respond primarily in a narrow frequency band, the absolute acceleration of the basemat was deemed sufficient to determine the state of the structure. As shown in Fig. 11, the reduction in the basemat response is greater than the reduction in the first-floor response. Theoretically, a sliding structure may be controlled by a collocated sensor

and actuator pair at the base, because the structure is excited only at the base. If the controller responds quickly and accurately, it can nullify the input; however, the large delay leads to an excitation in the superstructure before the controller may react. Subsequently, the controller has to control both the input and the interaction of the upper stories with the basemat. Under these conditions, feedback based on the

**Table 5**  
Experimental peak responses of rigid configuration

Earthquake (1)	Table Accel. (g) (2)	Basemat acceleration (g)			Basemat relative displacement (mm)			Control force (% of weight) (9)	Mapping factors	
		Hybrid (3)	Passive (4)	Ratio (5)	Hybrid (6)	Passive (7)	Ratio (8)		Input (10)	Output (11)
<i>Piecewise linear control profile</i>										
El Centro	0.30	0.060	0.085	71%	21.6	17.8	121%	8.61%	0.43	3.1
Taft	0.33	0.072	0.084	86%	20.8	19.3	107%	8.40%	0.43	3.1
Pacoima	0.34	0.057	0.098	58%	19.8	25.4	77%	6.07%	0.50	2.0
<i>Smooth nonlinear control profile</i>										
El Centro	0.30	0.065	0.085	76%	16.0	17.8	89%	6.01%	0.17	4.5
Taft	0.32	0.066	0.084	78%	21.3	19.3	110%	5.71%	0.17	4.5
Pacoima	0.34	0.060	0.098	61%	20.5	25.4	80%	6.84%	0.30	3.3
Hachinohe	0.21	0.065	0.079	82%	14.5	13.5	107%	4.58%	0.13	5.0
White noise	0.26	0.067	0.062	108%	10.9	6.9	159%	6.01%	—	—

absolute acceleration of the basemat alone may not be sufficient. The total shear force is a more appropriate feedback variable. The base shear force at the sliding interface was used for controlling the case of the basemat with the 3-DOF structure. The total shear force per structure weight was used for control

$$V_s = \frac{\sum_{k=0}^3 m_k(a_k)}{W} \quad (22)$$

where  $m_k$  and  $a_k$  are the mass and acceleration of the  $k$ th floor, and  $W$  is the total weight of the system.  $V_s$  is the normalized shear force, which is used as the input to the SISO controller. The absolute acceleration of the basemat is a direct measure of the shear force at the isolation interface. The results of an experiment with the flexible 3-DOF structure are shown in Fig. 12. The absolute acceleration of the basemat and the second story were reduced; however, the displacement responses were not significantly affected.

The fuzzy controller was effective in reducing the peak responses as compared with the passive system. Unfortunately, the control was less efficient at other points in the time history, as shown Figs. 10, 11, and 12. This occurred because the mapping factors were based on the peak responses and maintained constant for the whole time history. The negative effect of the large values of the mapping factors was accentuated by the time delay. Improvement of the response might be obtained if an adaptive system can be

developed that can modify the mapping factors, based on a robust performance criteria and continuous monitoring of the response.

## 7 DISCUSSION AND CONCLUSIONS

This study presents one of the first experimental applications of fuzzy control to a building structure. The study was performed analytically and experimentally. Several observations and conclusions are due from the presented material. (1) The experimental study has proven the feasibility of processing sufficient information, in real time, to implement the fuzzy algorithms in applications requiring the large control forces customary in civil engineering structures. (2) The study identified mapping factors, which act as control gains, that can be tuned to obtain improved performance. (3) The controller can be designed to simulate nonlinear control laws through fuzzy relations implemented with linguistic rules. (4) The controller can handle noisy inputs by providing automatic filtering in the fuzzification and defuzzification processes. The use of fuzzy sets allows handling of uncertainties in relations and input. (5) The actuator dynamics and delay are critical factors in determining the response of systems. Suitable compensation in the fuzzy terms and rules is necessary. (6) Difficulties in the implementation of acceleration control can be overcome by extending the system to either force control or a combination

**Table 6**  
Analytical versus experimental results for smooth nonlinear control profile

Earthquake motion (PGA) (1)	Control application (2)	Absolute accel. (gs) (3)	Relative displ. (mm) (4)	Control force (% weight) (5)	Mapping factors		
					Input (6)	Output (7)	Product (8)
El Centro (0.34g)	Analytical						
	No delay	0.060	27.18	4.86	0.28	2.11	0.59
	With delay	0.070	22.36	3.20	0.18	1.40	0.25
Taft (0.32g)	Experimental	0.065	15.98	6.01	0.17	4.50	0.77
	Analytical						
	No delay	0.065	30.21	5.04	0.29	2.19	0.64
Pacoima dam (0.4g)	With delay	0.075	25.19	3.10	0.15	1.36	0.20
	Experimental	0.066	21.33	5.71	0.17	4.50	0.77
	Analytical						
Hachinohe (0.23g)	No delay	0.060	25.22	4.63	0.27	2.01	0.54
	With delay	0.074	25.99	3.36	0.18	1.46	0.26
	Experimental	0.060	20.47	6.84	0.30	3.30	0.99
Hachinohe (0.23g)	Analytical						
	No delay	0.059	24.33	4.51	0.26	1.96	0.51
	With delay	0.064	18.92	3.10	0.19	1.34	0.26
Hachinohe (0.23g)	Experimental	0.065	14.78	4.58	0.13	5.00	0.65

of simultaneous acceleration, velocity, and displacement control. The increase in computational effort is not significant to cause a deterioration in the response. (7) Since the reduction in the absolute acceleration and the relative displacement of the base are contradictory in the case of a passive sliding system, both quantities have to be considered in the design of the controller. Relative displacement can be introduced as a second feedback so that the control algorithm can optimize the reduction of both responses. Such an extension is feasible without substantial effort. (8) Further studies are required to develop adaptive techniques for the automatic determination of control rules, such as a self-organizing controller based on neural networks.

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