

# CONTROL OF SLIDING-ISOLATED BRIDGE WITH ABSOLUTE ACCELERATION FEEDBACK

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**ABSTRACT:** This paper presents an experimental and analytical study of hybrid control of bridges using sliding bearings, with recentering springs, in parallel with servohydraulic actuators. A new control algorithm with absolute acceleration feedback, based on instantaneous optimal control laws, is developed. The developed control algorithm is implemented in a shake-table study of an actively controlled sliding-isolated bridge. The objective of implementing the hybrid system is to evaluate its advantages in addition to those due to the passive sliding system. The experimental system used in the shake-table test is described and the results of the experiments are presented. It is shown that substantial reduction of response acceleration is possible, using hybrid control, while confining the sliding displacement within an acceptable range, and eliminating almost completely postearthquake permanent offsets. Comparisons of the results of hybrid system with results of passive system are presented. The advantages of hybrid control, in addition to those due to passive control, are discussed.

## INTRODUCTION

Active control of linear and nonlinear structures, by means of active mass dampers, active tendon systems, active bracing systems, and variable stiffness/damping bracing, has been developed and tested in the laboratory (Chong et al. 1990; *Proc.* 1991; Kobori 1991; Reinhorn and Manolis 1985; Soong et al. 1991a; Yang and Soong 1988). Full-scale implementation of active control in prototype structures has been accomplished (Kobori 1991; Reinhorn et al. 1993; Soong et al. 1990, 1991b). Passive systems, in the form of base-isolation systems, both elastomeric and sliding-isolation systems, have found widespread application. Combined active and passive control systems, referred to as hybrid control systems, in which active mass dampers or actuators in parallel with base-isolation systems are employed, have been developed recently and shown to be effective (Feng et al. 1993; Kelly et al. 1992; Nagarajaiah et al. 1992; Reinhorn et al. 1987; Riley et al. 1991; Subramaniam et al. 1992; Tadjbakhsh et al. 1992; Yang et al. 1992a).

Sliding-isolation systems in bridges reduce the deck response acceleration and limit the damage to piers supporting the bridge deck. However, in the event of large earthquakes larger forces can be transferred, due to excessive sliding displacements, engagement of displacement restraints, and due to permanent offsets. One of the ways to effectively tackle these issues is to use combined active and passive control systems, referred to as hybrid control systems (Feng et al. 1991; Kelly et al. 1992; Nagarajaiah et al. 1993;

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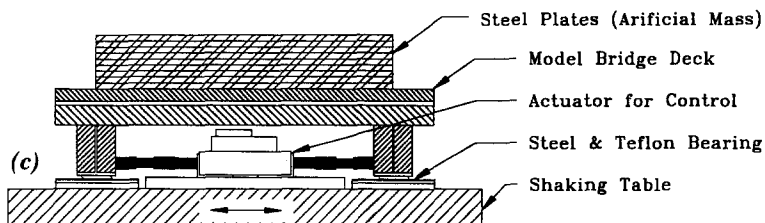
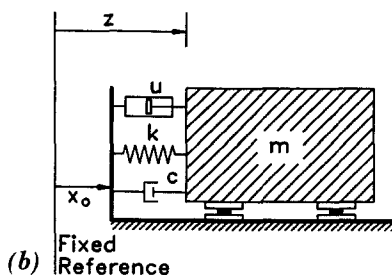
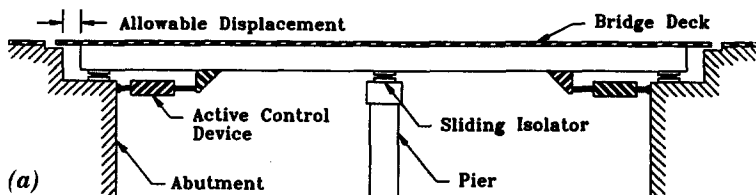
Note. Discussion open until April 1, 1994. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on September 28, 1992. This paper is part of the *Journal of Engineering Mechanics*, Vol. 119, No. 11, November, 1993. ©ASCE, ISSN 0733-9399/93/0011-2317/\$1.00 + \$.15 per page. Paper No. 4858.

Riley et al. 1992; Subramaniam et al. 1992; Tadjbakhsh et al. 1992; Yang et al. 1991). In the case of hybrid control the feedback state variables can be used to sense any increase in response acceleration. The feedback can then be used, with the appropriate gain factors, to apply control forces, which reduce the response acceleration to acceptable levels and control the displacements within acceptable limits. Additional advantages of hybrid isolation systems, when used in bridges (Feng et al. 1991; Nagarajaiah et al. 1992; Riley et al. 1992), are that: (1) They minimize the response acceleration in both large and small-scale earthquakes; (2) they limit the sliding displacements, so that displacement restraints are not engaged; and (3) they are more reliable than a purely active system, since the sliding isolators will still protect the structure from serious damage if the active portion were to fail.

A number of hybrid systems combining active control and base-isolation systems have been proposed to alleviate the problems just discussed. Previous hybrid designs include use of active mass dampers or actuators with elastomeric and sliding-isolation systems (Kelly et al. 1992; Nagarajaiah et al. 1992; Riley et al. 1991; Subramaniam et al. 1992; Tadjbakhsh et al. 1992; Yang et al. 1991, 1992a); use of active tendons or pulse generators with elastomeric bearing isolation systems (Reinhorn et al. 1987); and use of friction controllable sliding isolation system (Feng et al. 1993). The essential strategy adopted in most of these studies is to minimize the transfer of force through the isolation system by the use of either active mass dampers or actuators in parallel with the isolation system and by application of external forces opposing the forces in the isolation system. The friction-controllable sliding-isolation system developed by Feng et al. (1991) consists of friction-controllable sliding bearings, in which a fluid chamber inside the bearing effectively controls the bearing pressure at the sliding interface and in-turn controls the level of friction at the sliding interface. Shake-table tests (Feng et al. 1993; Nagarajaiah et al. 1992; Riley et al. 1992) have demonstrated the effectiveness and advantages of using such hybrid systems in bridges for limiting the transfer of seismic force to a minimum and confining the sliding displacement within an acceptable range.

This paper presents an experimental and analytical study of hybrid control of bridges using sliding bearings, with recentering springs, in parallel with servohydraulic actuators [see Fig. 1(a)]. A new control algorithm for hybrid systems is developed with absolute acceleration feedback. Instantaneous optimal control algorithms for hybrid control of nonlinear and hysteretic structures have been developed by Yang et al. (1992a, 1991) using relative acceleration feedback. Two main reasons lead to the use of relative acceleration feedback: (1) The performance of relative acceleration feedback is better than the displacement feedback; and (2) it is easier to measure acceleration response than displacement response. However, attempts to implement the instantaneous optimal control algorithm with relative acceleration feedback in the shake-table study of hybrid controlled sliding-isolated bridge described in this paper were not successful. Hence a new control algorithm with absolute acceleration feedback, referred to as absolute acceleration control algorithm, is developed and implemented in the study. Absolute acceleration is a more appropriate parameter for feedback since it represents the inertial force on the bridge deck; hence, control forces, based on absolute acceleration feedback, will oppose the inertial forces at any given instant.

The experimental setup and the control system used in the shake-table



**FIG. 1. Bridge with Hybrid Isolation System: (a) Two-Span Bridge; (b) Single-Degree-of-Freedom Model; (c) Bridge Model for Shake-Table Test**

test are described, and the results of the experiments are presented. It is shown that substantial reduction of response acceleration is possible using hybrid control, while confining the sliding displacement to within an acceptable range. Comparisons of the results of hybrid system with results of passive system are presented. The advantages of hybrid control, in addition to those due to passive control, are discussed.

### **ABSOLUTE ACCELERATION CONTROL ALGORITHM**

The control algorithm, with absolute acceleration feedback, developed herein is based on the instantaneous optimal control algorithm developed by Yang et al. (1991, 1992a).

A two-span bridge with hybrid isolation system is shown in Fig. 1(a). The motion of the bridge in the lateral direction is considered. Considering the piers, the abutments, and the bridge deck to be rigid, the bridge can be modeled as a rigid-mass single-degree-of-freedom (SDOF) system subjected

to earthquake excitation as shown in Fig. 1(b). The equation of motion of the SDOF system in absolute coordinate system is as follows:

$$m\ddot{z} + c(\dot{z} - \dot{x}_0) + k(z - x_0) + F_f = u \quad \dots\dots\dots (1)$$

where  $m$  = mass of the bridge;  $c$  = damping;  $k$  = stiffness of the recentering spring;  $u$  = control force;  $F_f$  = friction force;  $z$  = absolute displacement of the bridge deck; and  $x_0$  = absolute displacement of the ground.

The friction force can be represented by the following equation:

$$F_f = \mu mg \cdot \text{sgn}(\dot{z} - \dot{x}_0) \quad \dots\dots\dots (2)$$

where  $\mu$  = coefficient of friction;  $g$  = acceleration due to gravity; and  $\text{sgn}$  = signum function.

In this study the stick-slip phases are accounted for by using a hysteretic model for sliding interfaces (Constantinou et al. 1990). The model is based on the following differential equation proposed by Wen (1989):

$$\dot{y} = \{(\dot{z} - \dot{x}_0) - \gamma(\dot{z} - \dot{x}_0)y^2 \text{sgn}[(\dot{z} - \dot{x}_0)y] - \beta(\dot{z} - \dot{x}_0)y^2\} \frac{1}{Y} \quad \dots\dots (3)$$

where  $\text{sgn}$  = signum function;  $y$  = hysteretic dimensionless parameter;  $\gamma$  and  $\beta$  = dimensionless parameters; and  $Y$  = shear displacement of Teflon interface before sliding commences.

The friction force is represented as follows:

$$F_f = \mu mg \cdot y \quad \dots\dots\dots (4)$$

where  $\mu$  = coefficient of friction;  $g$  = acceleration due to gravity; and  $y$  = hysteretic dimensionless parameter, bounded by  $\pm 1$ , which accounts for stick-slip conditions instead of the signum function.

Eq. (1) can be reduced to two first-order differential equations

$$\dot{z}_1 = z_2 \quad \dots\dots\dots (5a)$$

$$\dot{z}_2 = -\omega^2 z_1 - 2\xi\omega z_2 - \mu g y + \omega^2 x_0 + 2\xi\omega \dot{x}_0 + \frac{u}{m} \quad \dots\dots\dots (5b)$$

where  $\omega$  = natural frequency of the SDOF system, neglecting the friction force; and  $\xi$  = damping ratio.

The state variables in (3) and (5) can be combined in the vector

$$\mathbf{z}(t) = \begin{Bmatrix} z_1 \\ z_2 \\ y \end{Bmatrix} \quad \dots\dots\dots (6)$$

The control force is determined by minimizing the following time-dependent objective function, proposed by Yang et al. (1992a, 1991), at every time instant  $t$  for the entire duration of an earthquake:

$$J(t) = \dot{\mathbf{z}}^T(t)\mathbf{Q}\dot{\mathbf{z}}(t) + Ru^2(t) \quad \dots\dots\dots (7)$$

in which  $\mathbf{Q}$  = a ( $3 \times 3$ ) positive semidefinite weighting matrix, and the weighting coefficient  $R$  is positive.

Eqs. (3), (5), and (6) can be represented by the following nonlinear matrix equation:

$$\dot{\mathbf{z}}(t) = \mathbf{g}[\mathbf{z}(t)] + \mathbf{b}u(t) + \mathbf{w}(t) \quad \dots\dots\dots (8)$$

where

$$\mathbf{g}[\mathbf{z}(t)] = \left\{ \begin{array}{c} z_2(t) \\ -\omega^2 z_1(t) - 2\xi\omega z_2(t) - \mu g y(t) \\ f(z_2(t), y(t), \dot{x}_0(t)) \end{array} \right\} \dots\dots\dots (9a)$$

$$\mathbf{b} = \left\{ \begin{array}{c} 0 \\ 1/m \\ 0 \end{array} \right\} \dots\dots\dots (9b)$$

$$\mathbf{w}(t) = \left\{ \begin{array}{c} 0 \\ \omega^2 x_0(t) + 2\xi\omega \dot{x}_0(t) \\ f(z_2(t), y(t), \dot{x}_0(t)) \end{array} \right\} \dots\dots\dots (9c)$$

Replacing the state vector  $\mathbf{z}(t)$  by its first forward finite-difference form [i.e.,  $\mathbf{z}(t) = \mathbf{z}(t - \Delta t) + \Delta t \dot{\mathbf{z}}(t - \Delta t)$ ] yields the following:

$$\dot{\mathbf{z}}(t) = \mathbf{g}[\mathbf{z}(t - \Delta t) + \Delta t \dot{\mathbf{z}}(t - \Delta t)] + \mathbf{b}u(t) + \mathbf{w}^*(t) \dots\dots\dots (10)$$

with zero initial conditions [i.e.,  $\dot{\mathbf{z}}(0) = \mathbf{z}(0) = 0$ ], and with

$$\mathbf{w}^*(t) = \left\{ \begin{array}{c} 0 \\ \omega^2 x_0(t) + 2\xi\omega \dot{x}_0(t) \\ f[z_2(t - \Delta t), y(t - \Delta t), \dot{x}_0(t)] \end{array} \right\} \dots\dots\dots (11)$$

It is to be noted that instead of the first forward difference, higher-order forward-difference expressions can be used for more accuracy.

To minimize the objective function  $J(t)$ , given by (7), subject to the constraint (10), the following Hamiltonian  $H$  is established, by introducing the Lagrangian multiplier vector  $\lambda$ :

$$H = \dot{\mathbf{z}}^T(t)\mathbf{Q}\dot{\mathbf{z}}(t) + Ru^2(t) + \boldsymbol{\lambda}^T\{\dot{\mathbf{z}}(t) - \mathbf{g}[\mathbf{z}(t - \Delta t) + \Delta t \dot{\mathbf{z}}(t - \Delta t)] - \mathbf{b}u(t) - \mathbf{w}^*(t)\} \dots\dots\dots (12)$$

The necessary conditions for minimizing the objective function  $J(t)$ , subject to the constraint of (10), are the following:

$$\frac{\partial H}{\partial \dot{\mathbf{z}}} = 0 \dots\dots\dots (13a)$$

$$\frac{\partial H}{\partial u} = 0 \dots\dots\dots (13b)$$

$$\frac{\partial H}{\partial \boldsymbol{\lambda}} = 0 \dots\dots\dots (13c)$$

Using closed-loop control the control vector,  $u(t)$ , is obtained by substituting (13) in (12), yielding the following:

$$u(t) = -\frac{1}{R} \mathbf{b}^T \mathbf{Q} \dot{\mathbf{z}}(t) \dots\dots\dots (14)$$

The response depends on the choice of weighting matrix  $\mathbf{Q}$  and weighting coefficient  $R$ . The weighting coefficient  $R = 1$  is chosen for simplicity. In the present case, only one element of the matrix  $\mathbf{Q}$  that is relevant is chosen to be nonzero, and that is  $Q(2,2) = 100.62$ ; i.e., the term corresponding

to absolute acceleration feedback. The analytical results are simulated numerically by integrating the equation of motion with the control force given by (14).

### EXPERIMENTAL SETUP

The tested single-span bridge model shown in Figs. 1(c) and 2 consisted of a steel bridge, with a total weight of 22.24 kN (5,000 lb), on four Teflon-steel sliding bearings. The recentering spring-hydraulic actuator connected between the shake table and the bridge deck is shown in Figs. 1(c) and 2.

The instrumentation consisted of seven accelerometers and displacement transducers used to monitor the lateral motions and twisting motions (see Fig. 2). One of these instruments was used both for control and for monitoring the system; the other six instruments were used solely for observation. The accelerometer attached to the bridge deck was used to measure the absolute accelerations. The control instrument was a  $\Delta p$  cell (see Fig. 2), which measured the pressure difference across the piston of the actuator. The  $\Delta p$  cell was connected in a closed loop to the analog controller, and used to control the force generated by the actuator. Two load cells were mounted at each end of the actuator piston (see Fig. 2). The signals from the two load cells were summed together and were used to measure the control force that was actually applied to the bridge deck.

A microcomputer was used to control the system. The relative displacement and velocity, and the absolute acceleration of the bridge deck were monitored by the computer that generated the control signal. The computer program calculated the control force based on (14), and also generated the recentering forces in the virtual spring used instead of an actual spring. The actual control signal was calculated as a change in pressure, which was sent to the  $\Delta p$  cell through an analog controller. A schematic of the control system is shown in Fig. 3.

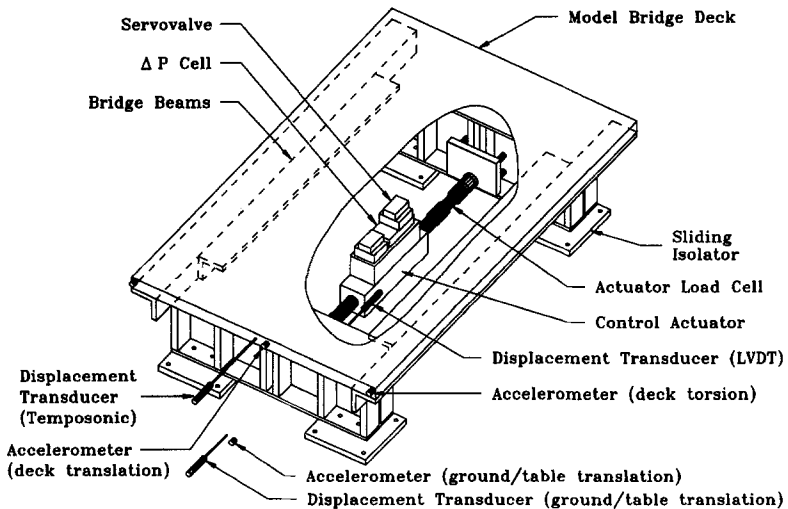


FIG. 2. Single-Span Bridge Model with Control System and Instrumentation Used in Shake-Table Test

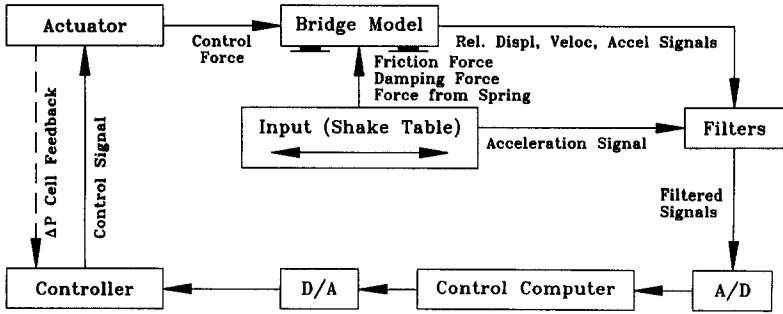


FIG. 3. Control System for Bridge Model

### ISOLATION SYSTEM

The four sliding bearings were made up of a Teflon-polished stainless steel interface. The measured coefficient of sliding friction followed the relationship proposed by Constantinou et al. (1990):

$$\mu = f_{\max} - \Delta f \cdot \exp(-a|z - \dot{x}_0|) \dots \dots \dots (15)$$

where  $f_{\max}$  = maximum coefficient of friction;  $\Delta f$  = difference between the maximum and minimum coefficients of friction; and  $a$  = constant. The measured values of  $f_{\max}$  varied between 0.17 and 0.22;  $\Delta f$  varied between 0.16 and 0.19; and  $a = 152.4$  mm/s (6 in./sec). Furthermore, the sliding bearings had an inclination of  $0.55^\circ$  in the testing direction when installed on the shake table.

The hydraulic actuator connected between the shake table and the bridge deck was capable of applying control forces of up to 24.46 kN (5,500 lb). The actuator and the bridge supports were connected with a 12.7 mm (1/2 in.) wire rope that was pretensioned to 11.12 kN (2,500 lb). The wire ropes were chosen because they allow for some out-of-plane motion in the system; a more rigid connection would not allow any such motion. A virtual recentering spring, implemented through the actuator, as mentioned earlier, was used instead of an actual recentering spring. A realistic isolation period of 2 sec ( $\omega = \pi$  rad/s), used in most prototype structures, was adopted.

### TEST PROGRAM

Four different sets of tests were conducted with  $Q(2,2) = 0, 100.62, 167.59, \text{ and } 217.88$ , to study the effect of weighting matrix  $Q$ . White-noise excitation was used to measure the time delay in the control system. The test program consisted of various earthquake input tests. Only a lateral earthquake excitation was applied, without any vertical components. The characteristics of the earthquakes used included predominantly low-frequency (Hachinohe) and moderately high frequency (Pacoima Dam) motions. The earthquake signals used were as follows:

1. San Fernando earthquake (Pacoima S16E), February 9, 1971; component S16E; peak ground acceleration (PGA) = 1.17g; predominant frequency range (PFR) = 0.25–6 Hz.

2. Tokachi-Oki earthquake (Hachinohe, Japan) of May 16, 1968; component EW; PGA = 0.2g; PFR = 0.25–2 Hz.
3. Artificial earthquake; PGA = 0.416g; PFR = 0.5–6 Hz.

## EXPERIMENTAL RESULTS

The experimental sequence included tests of hybrid control followed by tests of the passive system. The experimental results of the passive system had to be discarded because of an equipment malfunction. A summary of the experimental results of hybrid control tests for the case with  $Q(2,2) = 100.62$  is shown in Table 1. The time delay in the control system, which included the delay in the computer and the hydraulic system, was measured to be 48 ms in the white-noise tests. The system remained stable during white-noise excitation tests. No torsional motion was observed in the tests. Table 1 shows the excitation used in the test program, table acceleration, table displacement, deck acceleration, deck relative displacement, and control force. The earthquake excitations, shown in Table 1, were scaled to a percentage of the peak ground acceleration (PGA) of the actual record. For example, Pacoima Dam S16E 50% corresponds to the actual Pacoima Dam S16E record with the peak acceleration decreased approximately by a factor of 2. The input signals were not compensated for shaking-table errors. The accelerations of the bridge deck and the shake table, the absolute displacement of the shake table, and the relative displacement of the bridge deck with respect to the shake table, in the case of Pacoima Dam S16E 50% excitation and  $Q(2,2) = 100.62$ , are presented in Fig. 4.

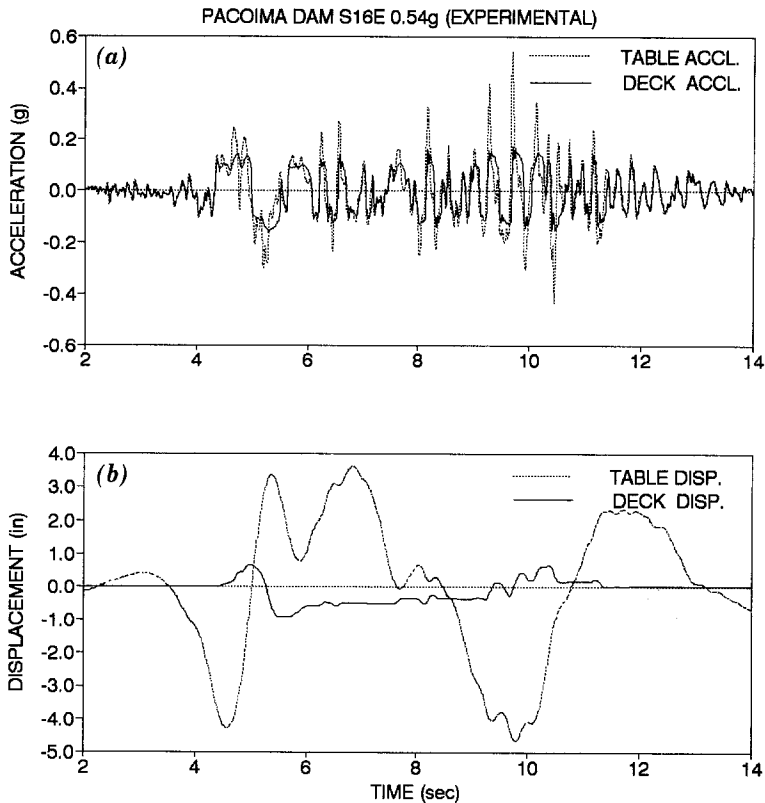
### Effectiveness of System

The results in Table 1 indicate that the hybrid isolation system is effective in reducing the peak deck acceleration by a factor of two to three when compared to the table acceleration. Furthermore, the hybrid system limits the relative displacement of the bridge deck to within 40% of the table displacement. The reduction of acceleration and displacement are clearly evident in Fig. 4; and also the permanent displacement is negligible. To evaluate the effect of active control on stick phases, or phases of high acceleration, the frequency-response function of acceleration (i.e., ratio of deck acceleration to shake-table acceleration) is examined in Fig. 5 [ $Q(2,2) = 100.62$ ]. Note the significant reduction in the frequency-response function

**TABLE 1. Peak Response of Bridge Model with Hybrid Control [ $Q(2,2) = 100.62$ ]**

Excitation (1)	Table acceleration (g) $\ddot{x}_0$ (2)	Table displacement (in.) $x_0$ (3)	PEAK BRIDGE DECK RESPONSE HYBRID CONTROL				
			Experimental			Analytical	
			Deck acceleration (g) $\ddot{z}$ (4)	Deck relative displacement (in.) $z - x_0$ (5)	Control force (lb) $u$ (6)	Deck acceleration (g) $\ddot{z}$ (7)	Deck relative displacement (in.) $z - x_0$ (8)
Pacoima Dam S16E 50%	0.540	4.640	0.177	0.875	560	0.166	0.965
Hachinohe EW 100%	0.195	3.685	0.134	0.766	582	0.129	0.812
Artificial motion	0.416	3.488	0.175	1.400	558	0.183	1.549



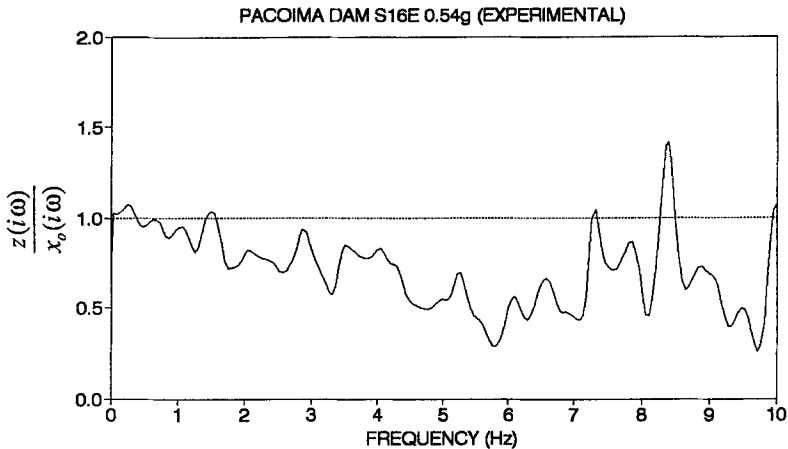


**FIG. 4. Measured Response of Bridge Model with Hybrid Isolation System Subjected to Pacoima Dam S16E 0.54g Excitation [ $Q(2,2) = 100.62$ ]; Responses Shown Are: Acceleration of Shake Table and Bridge Deck, Absolute Displacement of Shake Table, and Relative Displacement of Bridge Deck with Respect to Shake Table**

of acceleration, and hence the seismic force transferred, over a broad frequency range. The reduction is larger in the high-frequency region (4–10 Hz). The high frequency–high acceleration generated by stick phases is the essential cause for large seismic force transfer. The reduction, in Fig. 5, in the high-frequency region indicates the effectiveness of the hybrid system in reducing the stick phases and in ensuring continuous sliding. In essence the hybrid system complements the sliding-isolation system by ensuring continuous sliding and preventing stick phases i.e., phases of high acceleration. Also it is evident from Table 1 that the reduction occurs regardless of the frequency content of the earthquake. The control force needed is within 12% of the weight of the bridge, indicating the efficiency of the system. Results for a different set of tests and earthquakes can be found in Nagarajaiah et al. (1992).

#### **Effect of Increasing Control Force**

The control force given in (14) is proportional to the absolute acceleration of the bridge deck. Absolute acceleration represents the inertial force on



**FIG. 5. Frequency-Response Function of Deck Acceleration to Shake-Table Acceleration under Pacoima Dam S16E 0.54g Excitation [ $Q(2,2) = 100.62$ ]**

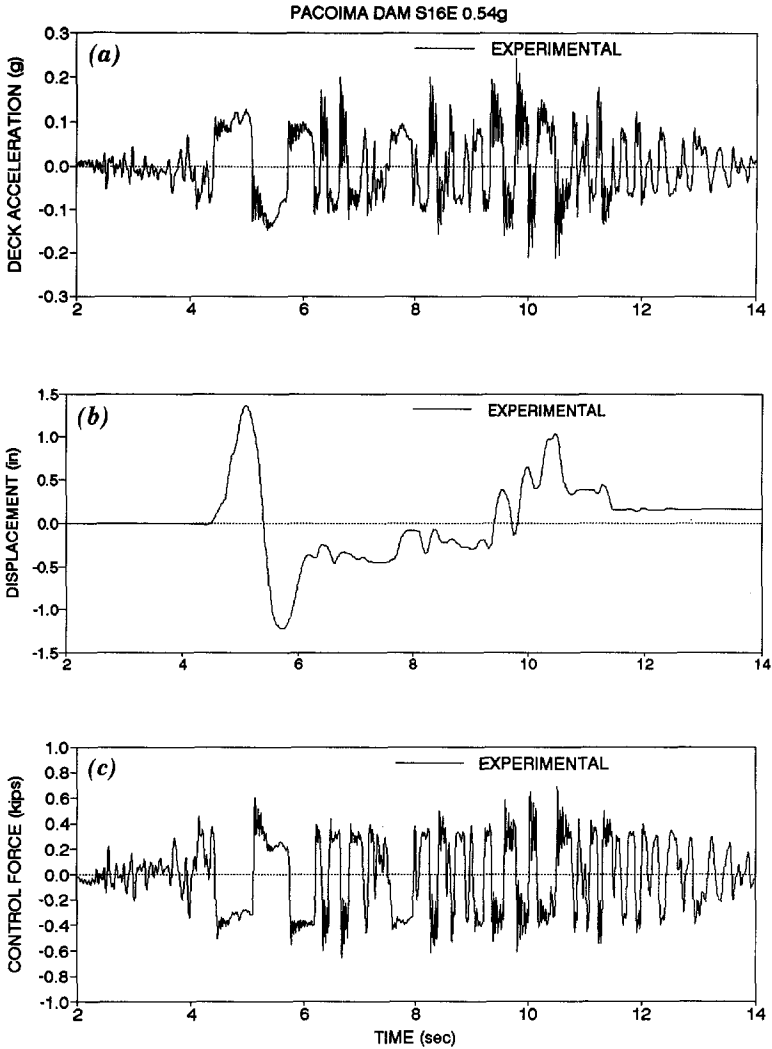
the bridge deck; hence, the value of weighting matrix,  $\mathbf{Q}$ , determines the level of control force that opposes the inertial force at any given instant. In theory, larger values of  $\mathbf{Q}$  will negate the inertial force to a larger extent, and in the limit, the inertial force will reduce to zero. However, in practice the time delay and inaccuracies in the system lead to errors in the control that will increase with increasing values of  $\mathbf{Q}$ .

As mentioned before, three sets of hybrid control tests were performed with different values [ $Q(2,2) = 100.62, 167.59, \text{ and } 217.88$ ], which resulted in increased control forces. The results for the case of Pacoima Dam S16E 50% excitation and  $Q(2,2) = 167.59$  are shown in Fig. 6. From Fig. 6 it is evident that the effectiveness of hybrid control was reduced in this case, when compared to the case with  $Q(2,2) = 100.62$ , even though a larger control force was applied. The deck-acceleration time history and the control-force time history also indicate that the deterioration in hybrid control resulted in instability after stick events, causing the system to exhibit unstable oscillations before stabilizing again. When the value of  $Q(2,2)$  was increased to 217.88 the system became unstable. However, the system was stable for all earthquakes considered in this paper and in Nagarajaiah et al. (1992) and white-noise excitation, when  $Q(2,2) = 100.62$ . The aforementioned observations indicate that the choice of the weighting matrix  $\mathbf{Q}$  is critical (Yang et al. 1992b).

Analytical investigations incorporating the time-delay effect approximately by using the absolute acceleration feedback from time  $t - \tau$  ( $\tau =$  time delay) instead of time  $t$  indicate that the reasons for the observed instability are mainly the time delay and larger gain factors. Similar observations have been made by Agrawal (1991). An estimate of the weighting matrix  $\mathbf{Q}$  for which stable operations are possible can be obtained by incorporating the time-delay effect.

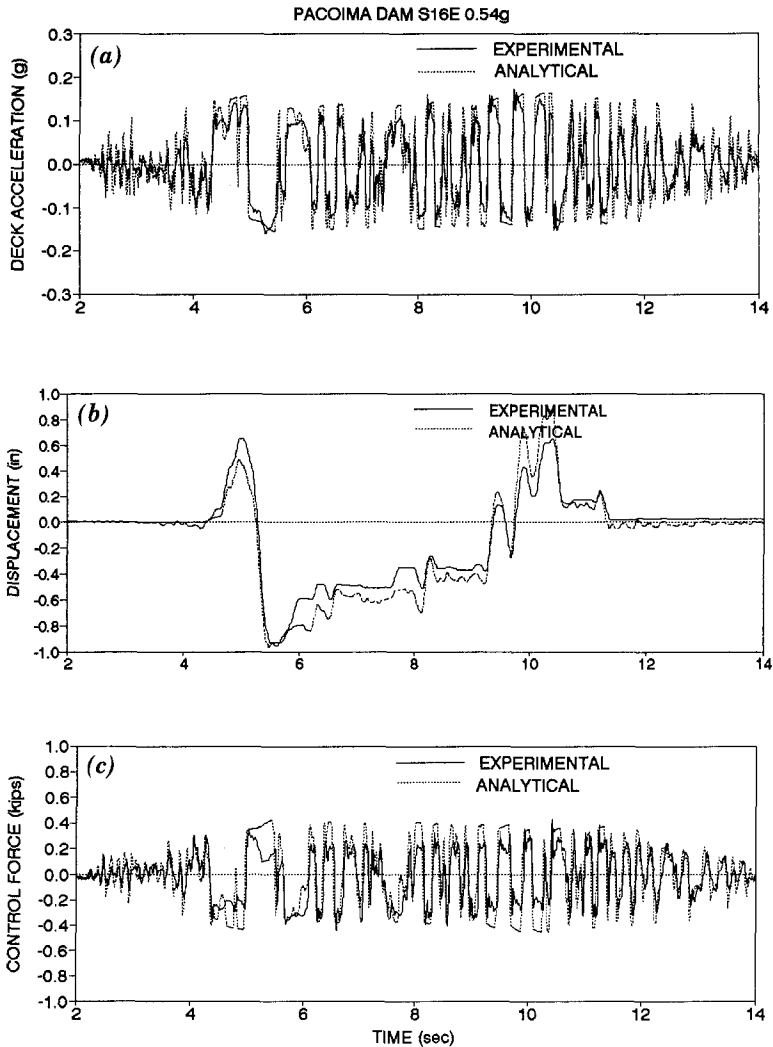
### **COMPARISON OF HYBRID CONTROL AND PASSIVE SYSTEM RESULTS**

The hybrid control results are numerically simulated using  $Y = 0.254$  mm (0.01 in.),  $\gamma = 0.9$ , and  $\beta = 0.1$  in (3); and  $\omega = \pi$  rad/s and  $\xi =$



**FIG. 6. Measured Response of Bridge Model with Hybrid Isolation System Subjected to Pacoima Dam S16E 0.54g Excitation [ $Q(2,2) = 167.59$ ]**

0.001, to evaluate the accuracy of the analytical model. A comparison of the experimental and analytical results in the hybrid control case is shown in Fig. 7 and Table 1 [ $Q(2,2) = 100.62$ ]. From Fig. 7 and Table 1 it is evident that the analytical model is capable of capturing the response satisfactorily, despite the fact that the control algorithm was developed using Coulomb friction instead of velocity-dependent friction. However, the differences in the measured-control-force time history and the computed-control-force time history indicate that the control implementation involved time delay and actuator dynamics. Since analytical results are satisfactory the analytical model is used to simulate the results of the passive case by setting  $Q(2,2) = 0.0$  (i.e., uncontrolled case). For further comparison of



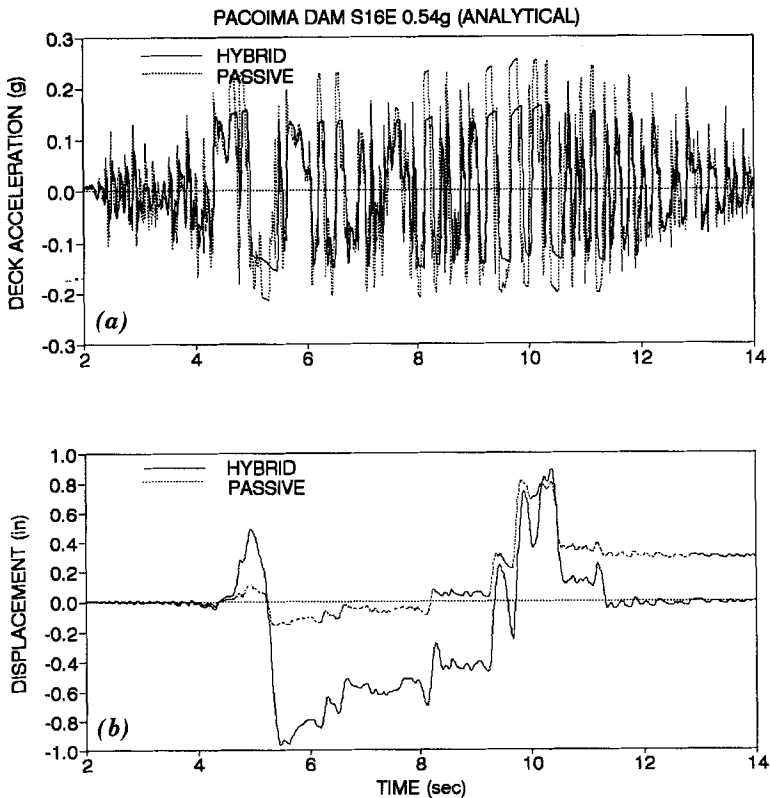
**FIG. 7. Comparison of Measured and Computed Response of Bridge Model with Hybrid Isolation System Subjected to Pacoima Dam S16E 0.54g Excitation [ $Q(2,2) = 100.62$ ]; Responses Shown Are Deck Acceleration, Deck Relative Displacement with Respect to Shake Table, Control Force**

hybrid control response and passive system response analytical results are used in lieu of the discarded experimental data.

The analytical results of the hybrid control case are compared with the analytical results of the passive case in Table 2 for various earthquakes and in Fig. 8 for Pacoima Dam S16E 50% [ $Q(2,2) = 100.62$ ]. From the results in Table 2 (columns 4–8) and Fig. 8 it is evident that in the case of Pacoima Dam S16E 50% excitation additional reductions in acceleration of up to 35% occur in the hybrid system, over and above the reductions due to the

**TABLE 2. Computed Peak Response of Bridge Model with Hybrid Control and Passive System [ $Q(2,2) = 100.62$ ]**

Excitation (1)	Table accelera- tion (g) $x_0$ (2)	Table displace- ment (in.) $x_0$ (3)	Hybrid Control (High Friction)		Passive System (High Friction)		Passive System (Low Friction)	
			Deck accelera- tion (g) $\ddot{z}_0$ (4)	Deck relative displace- ment (in.) $z - x_0$ (5)	Deck accelera- tion (g) $\ddot{z}_0$ (6)	Deck relative displace- ment (in.) $z - x_0$ (7)	Deck accelera- tion (g) $\ddot{z}$ (8)	Deck relative displace- ment (in.) $z - x_0$ (9)
			Pacoima Dam S16E 50%	0.540	4.640	0.166	0.965	0.254
Hachinohe EW 100%	0.195	3.740	0.129	0.812	0.180	0.180	0.140	1.396
Artificial motion	0.416	3.488	0.183	1.549	0.230	0.640	0.181	3.265



**FIG. 8. Comparison of Computed Response of Bridge Model with Hybrid [ $Q(2,2) = 100.62$ ] and Passive Isolation Systems Subjected to Pacoima Dam S16E 0.54g Excitation (Compare with Fig. 7)**

passive system, while the sliding displacement is within an acceptable range when compared to the passive case. Also, note the smaller permanent displacement in the hybrid system (see Fig. 8). Further results in Table 2 indicate that reduction in response occurs both in large earthquakes like the Pacoima Dam S16E and moderate earthquakes like Hachinohe EW. Further comparisons for a different set of tests and earthquakes can be found in Nagarajaiah et al. (1992).

One can argue that the reduction in acceleration achieved in the case of hybrid system can also be achieved by a passive system by using sliding bearings with a lower coefficient of friction. To resolve this issue, the responses of passive systems with lower coefficient of friction are examined in Table 2 (columns 8 and 9). Passive systems with a lower coefficient of friction  $f_{\max}$  and coefficient  $\Delta f$ , in (15), that yield an acceleration approximately equal to the hybrid case are found by trial and error. The coefficients  $f_{\max}$  and  $\Delta f$  found by trial and error are: (1) 0.12 and 0.11 in the case of Pacoima Dam S16E 50%; (2) 0.09 and 0.08 in the case of Hachinohe EW 100%; and (3) 0.07 and 0.06 in the case of artificial motion, respectively. The constant  $a = 152.4$  mm/s (6 in./sec) is identical in all the cases. The results shown in Table 2 (columns 8 and 9) indicate that the reduction in acceleration can be achieved by a passive system with sliding bearings having a lower coefficient of friction; however, this is achieved at the expense of much larger sliding displacements, than in the hybrid case, and larger permanent offsets.

## CONCLUSIONS

A new control algorithm, with absolute acceleration feedback, based on instantaneous optimal control, was developed. The new control algorithm was implemented in a shake-table study. The main conclusions drawn from the experimental and analytical results are as follows.

The hybrid system complements the sliding-isolation system by ensuring continuous sliding and preventing stick conditions; i.e., phases of high frequency-high acceleration response. Significant reductions of peak deck acceleration response over a broad frequency range occurs because of the hybrid system, while the sliding displacements are maintained within an acceptable range. The reduction occurs regardless of the frequency content of the earthquake and for both large and moderate earthquakes alike. The control force needed is within 12% of the weight of the bridge indicating the efficiency of the hybrid system. The permanent offsets are either negligible or the bridge can easily be recentered by the use of control actuators.

The system is stable for all the excitations when  $Q(2,2) = 100.62$ . However, increasing levels of control force [i.e.,  $Q(2,2) = 167.59$  and  $217.88$ ] causes instability. The reasons for this instability, which are mainly the time delay and larger gain factors, are the subject of a separate study. The choice of the weighting matrix  $Q$  is critical in avoiding instability.

Comparison between the hybrid control and passive system responses indicate that the hybrid system reduces the acceleration response further, while reducing the sliding displacement, when compared to the passive case. A reduction of deck acceleration can also be achieved by a passive system with sliding bearings having lower coefficient of friction; however, this can be achieved only at the expense of much larger sliding displacements and permanent offsets.

The response of the system can be reliably predicted using the presented analytical model.

The effectiveness of hybrid isolation system for bridges was presented in this paper. However, the application of the hybrid isolation system is not restricted to bridges and can also be applied to buildings, floor isolation systems and to secondary systems.

## ACKNOWLEDGMENTS

The writers are grateful to Professor Masanobu Shinozuka for discussions and valuable suggestions during the course of this study. The writers wish to acknowledge the partial financial support provided the National Center for Earthquake Engineering Research (grant NCEER 915111) for this study. Acknowledgments are also due to M. Pitman and D. Walch for technical support in the laboratory.

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