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Linear-Matrix-Inequality-Based Robust Fault Detection and Isolation Using the Eigenstructure Assignment Method

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Nomenclature

A	=	system state transmission matrix with dimension $n \times n$
B_d	=	noise input influence matrix with dimension $n \times n_d$
B_u	=	input influence matrix with dimension $n \times r$
C	=	output influence matrix with dimension $m \times n$
D_d	=	noise direct transmission matrix with dimension $m \times n_d$
F_i	=	ith force direction vector
H	=	projection matrix
$\mathbf{I}$	=	identity matrix
L	=	observer gain with dimension $n \times m$
m	=	number of outputs
m_i	=	arbitrary scalar function of time with respect to the ith fault direction
n	=	number of states
n_d	=	number of disturbance inputs
q	=	number of faults
r	=	number of inputs

Introduction

A FAULT is defined as any kind of malfunction in the actual physical system that tends to degrade the overall system performance. Such malfunction may occur either in the sensors (instruments), actuators, or in the components of the process [1]. To maintain a high level of performance, it is important that failures can be promptly detected and identified so that appropriate remedies can be applied.

In the past decades, numerous approaches to the problem of failure detection, isolation, and accommodation (FDIA) [1] in dynamic systems have been developed. Among them are two major FDI philosophies: physical redundancy and analytical redundancy. Physical redundancy is achieved simply through hardware replication. Unlike physical redundancy, analytical redundancy, which implies the inherent redundancy contained in the static and dynamic relationship among the system inputs and measured outputs [1], is a model-based method and has recently gained increasing attention. Analytical redundancy methods have many advantages over physical redundancy methods; for example, the replication of identical hardware components (actuator/sensor) is more expensive, restricted, and sometimes difficult to implement in practice [2,3]. There are many analytical redundancy-based FDI methods. Beard–Jones detection filters (BJDT) are one of the most popular FDI methods used today. In their pioneering work done in the early seventies, Beard [4] and Jones [5] found that with the proper choice of filter feedback gains, the filter residual will have directional characteristics that can be easily associated with different faults. The BJDT filters were enhanced later by many people [6–11]. Among them, the eigenstructure assignment method has special advantages, because the stability and dynamic behavior of a linear multivariable system are governed by the eigenstructure of the system [7,9,11,12].

The BJDT filters rely on the idealized mathematical model of the system. In practice, this assumption can never be perfectly satisfied, because measurement noise and modeling error always exist. Therefore, robust filter design has become more and more important. Douglas and Speyer [10] developed an algorithm to compute the H_∞-bounded fault detection filter. However, the presented modified algebraic Riccati equation has no associated Hamiltonian, and so it is difficult to find necessary or sufficient conditions for when a solution exists [8]. Recently, the linear-matrix-inequality (LMI) technique has been widely used in H_∞ control design because of its computational efficiency [13,14].

There are existing studies on fault detection [15,16], but fault isolation is a more difficult task and more important goal in terms of damage assessment and prognosis. In this Note, fault detection and isolation are achieved by the eigenstructure assignment method and LMI technique. Especially, eigenstructure assignment is realized by the concepts of detection spaces. This idea is new and effective. Once a set of stable eigenvalues with respect to each independent detection space is assigned, the LMI technique can be used to find the optimal detection filter, so that the noise effect on the residual is reduced. Compared with the previous filters, the proposed FDI filter has good FDI performance as well as the following advantages:

1) The eigenvalues are assigned in the detection space, which is the sum of minimum (C, A) invariant subspace and invariant zero directions for F_i. It extends the applicability of the detection filter.

2) The designed filter is robust to disturbances, which makes the filter more practically useful.

3) The LMI technique is used to solve the optimal filter problem, which is computationally efficient. The proposed method can be used to perform real-time fault detection and isolation. The presented numerical example demonstrates this capability.

Observer-Based System Residual Output Generation

The state-space model of a linear time-invariant (LTI) dynamic system with additive faults and without disturbances in continuous-time format is

$$\dot{x}(t) = Ax(t) + B_u u(t) + \sum_{i=1}^{q} F_i m_i(t) \qquad y(t) = Cx(t) \tag{1}$$

where $x \in \mathcal{X}$, $y \in \mathcal{Y}$, $u \in \mathcal{U}$, and $F_i \in F_i$. When no faults occur, $m_i(t) = 0$. The fault direction F_i can be used to model actuator, sensor, and component faults [3,8]. The system is assumed to be

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observable. Considering the following full-order observer, we have the error system dynamics:

$$\dot{\varepsilon}(t) = (A + LC)\varepsilon(t) - \sum_{i=1}^{q} F_i m_i(t) \qquad r(t) = C\varepsilon(t) \tag{2}$$

where $r(t)$ is the residual output. Clearly, if L is chosen such that $(A + LC)$ is stable, $r(t)$ in the steady state will deviate from zero when any of faults has occurred. But it is not enough, because we also want to isolate different faults and that is a more difficult task. To detect and isolate faults, a set of detection subspaces $\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_q$ is assumed to satisfy the following three conditions:

$$(A + LC)\mathcal{T}_i \subseteq \mathcal{T}_i; \qquad \mathcal{F}_i \subseteq \mathcal{T}_i; \qquad C\mathcal{T}_i \cap \sum_{j\neq i}^{q} C\mathcal{T}_j = \emptyset \tag{3}$$

Then the residual $r(t)$ has a unique representation of $r(t) = z_1 + z_2 + \ldots + z_q$ with $z_i \in C\mathcal{T}_i$ [8]. Therefore, faults can be isolated by projecting $r(t)$ onto independent subspaces $C\mathcal{T}_i$. Douglas [8] proved that the detection subspace $\mathcal{T}_i$ can be spanned by the following linearly independent vectors:

$$T_i = [v_{\text{is}} \ \cdots \ v_{i1} \ F_i \ AF_i \ \cdots \ A^{k_i}F_i] \tag{4}$$

where k_i is the smallest integer such that $CA^{k_i}F_i \neq 0$, and z_{ik} and v_{ik} are the invariant zero and associated zero direction, which are defined as $(k = 1, 2, \ldots, s)$

$$\begin{cases} (A + F_i K)v_{ik} = z_{ik}v_{ik} \\ Cv_{ik} = 0 \end{cases} \tag{5}$$

Because $(A + LC)\mathcal{T}_i \subseteq \mathcal{T}_i$ and $A^{k_i}F$ is in the column space of T_i, we can define

$$(A + LC)A^{k_i}F_i = T_i x_i = T_i[\beta_{\text{is}} \ \cdots \ \beta_{i1} \ \alpha_0 \ \alpha_1 \ \cdots \ \alpha_{k_i}]^T \tag{6}$$

Set $E = [CA^{k_1}F_1 \ CA^{k_2}F_2 \ \cdots \ CA^{k_q}F_q]$ and $\bar{T}_i = [v_{\text{is}} \ \cdots \ v_{i1} \ F_i \ AF_i \ \cdots \ A^{k_i-1}F_i]$. Suppose E has full-column rank so that the pseudoinverse of E exists. Based on the definitions of v_{ik} and k_i, we have $C\bar{T}_i = 0$, for $i = 1, 2, \ldots, q$. Assume that there is a subspace $\hat{T}$ satisfying the following condition:

$$\bar{T}_1 \oplus \bar{T}_2 \oplus \cdots \oplus \bar{T}_q \oplus \hat{T} = \ker(E_c) \tag{7}$$

where $E_c = E^+C$, $E^+ = (E^TE)^{-1}E^T$. Here, $\hat{T}$ is named the complementary subspace. It can be proved that $T_1 \oplus T_2 \oplus \cdots \oplus T_q \oplus \hat{T} = R^n$. But if $T_1, \ldots$ and T_q already span the whole space, then $\hat{T}$ does not exist.

Detection Filter Design

Eigenvalues of $(A + LC)$

Rewrite Eq. (5) as $Av_{ik} = z_{ik}v_{ik} - F_i(Kv_{ik}) = z_{ik}v_{ik} + y_{ik}F_i$, where $y_{ik} = -Kv_{ik}$. Define $V = [T_1, T_2, \ldots, T_q, \hat{T}]$ and $U = [U_1^T, U_2^T, \ldots, U_q^T, \hat{U}]^T$ such that $UV = \mathbf{I}$. Based on the definition of k_i and Eq. (5), we can perform the similarity transformation on matrix $(A + LC)$ by using U and V:

$$U(A + LC)V = \begin{bmatrix} U_1 \\ \vdots \\ U_q \\ \hat{U} \end{bmatrix} (A + LC)[T_1 \ \cdots \ T_q \ \hat{T}] = \begin{bmatrix} Y_q & \cdots & 0 & U_1(A + LC)\hat{T} \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & Y_1 & U_q(A + LC)\hat{T} \\ 0 & \cdots & 0 & \hat{U}(A + LC)\hat{T} \end{bmatrix} \tag{8}$$

where $Y_i = U_i(A + LC)T_i$ for $i = 1, 2, \ldots, q$.

Because the similarity transformation on a matrix does not change eigenvalues of that matrix, eigenvalues of $(A + LC)$ are the sum of eigenvalues of matrices $Y_1, \ldots, Y_q$ and $\hat{U}(A + LC)\hat{T}$. The characteristic equation of Y_i is

$$|\lambda I - Y_i| = \begin{cases} \lambda^{k_i+1} - \alpha_{k_i}\lambda^{k_i} - \alpha_{k_i-1}\lambda^{k_i-1} - \cdots - \alpha_1\lambda - \alpha_0 = 0 & \text{when s} = 0 \\ A_{i0}(\lambda)(\lambda^{k_i+1} - \alpha_{k_i}\lambda^{k_i} - \alpha_{k_i-1}\lambda^{k_i-1} - \cdots - \alpha_1\lambda - \alpha_0) - \sum_{j=1}^{s}[A_{ij}(\lambda)y_{ij}\beta_{ij}] = 0 & \text{when s} \neq 0 \end{cases} \tag{9}$$

where

$$A_{i0}(\lambda) = \prod_{j=1}^{s}(\lambda - z_{ij})$$

and

$$A_{ij}(\lambda) = \prod_{l\neq j}^{s}(\lambda - z_{il})$$

Let $\lambda_1, \lambda_2, \ldots, \lambda_{k_i+s+1}$ be a set of complex numbers symmetric about the real axis but otherwise arbitrary and assigned to be eigenvalues of Y_i. Substituting them into Eq. (9), we obtain

$$\begin{bmatrix} A_0(\lambda_1)\lambda_1^{k_i} & A_0(\lambda_1)\lambda_1^{k_i-1} & \cdots & A_0(\lambda_1) & A_{i1}(\lambda_1)y_{i1} & \cdots & A_{iq}(\lambda_1)y_{is} \\ A_0(\lambda_2)\lambda_2^{k_i} & A_0(\lambda_2)\lambda_2^{k_i-1} & \cdots & A_0(\lambda_2) & A_{i1}(\lambda_2)y_{i1} & \cdots & A_{iq}(\lambda_2)y_{is} \\ A_0(\lambda_3)\lambda_3^{k_i} & A_0(\lambda_3)\lambda_3^{k_i-1} & \cdots & A_0(\lambda_3) & A_{i1}(\lambda_3)y_{i1} & \cdots & A_{iq}(\lambda_3)y_{is} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A_0(\lambda_{k_i+s+1})\lambda_{k_i+s+1}^{k_i} & A_0(\lambda_{k_i+s+1})\lambda_{k_i+s+1}^{k_i-1} & \cdots & A_0(\lambda_{k_i+s+1}) & A_{i1}(\lambda_{k_i+s+1})y_{i1} & \cdots & A_{iq}(\lambda_{k_i+s+1})y_{is} \end{bmatrix} \begin{Bmatrix} \alpha_{k_i} \\ \alpha_{k_i-1} \\ \vdots \\ \alpha_0 \\ \beta_{i1} \\ \vdots \\ \beta_{is} \end{Bmatrix} = \begin{Bmatrix} A_0(\lambda_1)\lambda_1^{k_i+1} \\ A_0(\lambda_2)\lambda_2^{k_i+1} \\ A_0(\lambda_3)\lambda_3^{k_i+1} \\ \vdots \\ A_0(\lambda_{k_i+s+1})\lambda_{k_i+s+1}^{k_i+1} \end{Bmatrix} \tag{10}$$

The unknowns parameters $\alpha_0, \ldots, \alpha_{k_i}$ and $\beta_{i1}, \ldots, \beta_{\mathrm{is}}$ can be solved from the preceding linear equations. These parameters are only related to the assigned eigenvalues of Y_i and can be used to compute Γ in Eq. (14). But the eigenvalues of $\hat{U}(A+LC)\hat{T}$ are determined by the observer gain L when $\hat{T} \not\subset \ker(C)$. It gives us the freedom to select L such that $\hat{U}(A+LC)\hat{T}$ is stable and the residual output is robust to noise. However, if $\hat{T} \subseteq \ker(C)$, then $\hat{U}(A+LC)\hat{T} = \hat{U}A\hat{T}$. The eigenvalues of $\hat{U}(A+LC)\hat{T}$ do not rely on L. In this case, the eigenvalues of $\hat{U}A\hat{T}$ are assumed to be stable to ensure the asymptotic stability of fault isolation.

Robust Detection Filter Design Using the LMI Approach

Consider a LTI dynamic system with additive faults and disturbances:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_u u(t) + B_d\,\mathrm{d}(t) + \sum_{i=1}^{q} F_i m_i(t) \\ y(t) &= Cx(t) + D_d\,\mathrm{d}(t) \end{aligned} \tag{11}$$

Similarly, the state estimation error dynamics can be described as

$$\dot{\varepsilon}(t) = (A+LC)\varepsilon(t) - (B_d + LD_d)\,\mathrm{d}(t) - \sum_{i=1}^{q} F_i m_i(t)$$

$$r(t) = C\varepsilon(t) - D_d\,\mathrm{d}(t) \qquad z(t) = Hr(t) = HC\varepsilon(t) - HD_d\,\mathrm{d}(t) \tag{12}$$

where $z(t)$ is the failure indicator.

An optimal observer gain L needs to be designed such that the noise effect on the failure indicator is reduced. Noticing Eq. (6), we have

$$(A+LC)[A^{k_1}F_1 \;\; A^{k_2}F_2 \;\; \cdots \;\; A^{k_q}F_q] = [T_1 \;\; T_2 \;\; \cdots \;\; T_q] \begin{bmatrix} x_1 & 0 & \cdots & 0 \\ 0 & x_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & x_q \end{bmatrix} \tag{13}$$

where $x_i (i = 1, 2, \ldots, q)$ can be solved by Eq. (10). Hence, the solution of L is

$$L = \Gamma E^+ + Z_1(I - EE^+) \tag{14}$$

where

$$E = [CA^{k_1}F_1 \;\; CA^{k_2}F_2 \;\; \cdots \;\; CA^{k_q}F_q]$$

$$\Gamma = [T_1 \;\; T_2 \;\; \cdots \;\; T_q] \begin{bmatrix} x_1 & 0 & \cdots & 0 \\ 0 & x_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & x_q \end{bmatrix} - [A^{k_1+1}F_1 \;\; A^{k_2+1}F_2 \;\; \cdots \;\; A^{k_q+1}F_q]$$

E^+ is the pseudoinverse of E, and Z_1 is the arbitrary matrix with compatible dimensions.

As shown in Eq. (3), the residual will lie in the subspace spanned by $[CT_1, CT_2, \ldots, CT_q]$ if the noise term is ignored. Based on the definition of T_i, $CT_i = [0, 0, \ldots, CA^{k_i}F_i]$. Therefore, the projection matrix H can be designed as

$$HE = \Lambda \tag{15}$$

where Λ is any diagonal matrix. Solving the preceding equation, we have

$$H = \Lambda E^+ + Z_2(I - EE^+) \tag{16}$$

where Z_2 is the arbitrary matrix with compatible dimensions.

There are only two unknown matrices: Z_1 and Z_2 in Eqs. (14) and (16). We are now free to select suitable Z_1 and Z_2 such that the noise effect on the failure indicator is reduced. This step can be achieved by LMI techniques. The following statement comes from the bounded real (BRL) Lemma in [13].

The H_∞ norm of the transfer function from disturbances $\mathrm{d}(t)$ to the failure filter indicator $z(t)$ is less than γ if, and only if, there is a matrix $P > 0$, Q, and Z_2 such that the following LMI is satisfied:

$$\begin{bmatrix} P(A+\Gamma E^{+}C)+(A+\Gamma E^{+}C)^{T}P \\ +Q(I-EE^{+})C+C^{T}(I-EE^{+})^{T}Q^{T} & -P(B_d+\Gamma E^{+}D_d)-Q(I-EE^{+})D_d & (\Lambda E^{+}C+Z_2(I-EE^{+})C)^{T} \\ -(B_d+\Gamma E^{+}D_d)^{T}P-D_d^{T}(I-EE^{+})^{T}Q^{T} & -\gamma I & -(\Lambda E^{+}D_d+Z_2(I-EE^{+})D_d)^{T} \\ \Lambda E^{+}C+Z_2(I-EE^{+})C & -\Lambda E^{+}D_d+Z_2(I-EE^{+})D_d & -\gamma I \end{bmatrix} < 0 \tag{17}$$

where $Z_1 = P^{-1}Q$.

Numerical Simulation

The following example reexamines a modified F16XL aircraft fault detection system [8–10]. A first-order Dryden wind gust model is included:

$$\dot{x}(t) = Ax(t) + B_{\delta}\delta(t) + B_{\omega}\omega(t) \qquad y(t) = Cx(t) + Dv(t) \tag{18}$$

where the system matrices are

$$A = \begin{bmatrix} -0.0674 & 0.0430 & -0.8886 & -0.5587 & 0.0430 \\ 0.0205 & -1.4666 & 16.5800 & -0.0299 & -1.4666 \\ 0.1377 & -1.6788 & -0.6819 & 0 & -1.6788 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.1948 \end{bmatrix}$$

$$B_{\omega} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1.57 \end{bmatrix}, \qquad B_{\delta} = \begin{bmatrix} -0.1672 \\ -1.5179 \\ -9.7842 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0.0591 & 0 & 0 & 0.0591 \\ 0.0139 & 1.0517 & 0.1485 & -0.0299 & 0 \\ -0.0677 & 0.0431 & 0.0171 & 0 & 0 \end{bmatrix} \quad \text{and}$$

$$D = \begin{bmatrix} 0.01 & 0 & 0 & 0 \\ 0 & 0.143 & 0 & 0 \\ 0 & 0 & 0.245 & 0 \\ 0 & 0 & 0 & 0.245 \end{bmatrix}$$

Now two faults are considered: a normal accelerometer sensor fault and an elevon fault. The normal acceleration sensor fault can be modeled as an additive term in the measurement equation:

$$y(t) = Cx(t) + E_{Az}\mu_{Az}(t) \tag{19}$$

where $E_{Az} = [0\ \ 0\ \ 1\ \ 0]^{T}$, and μ_{Az} is an arbitrary time-varying real scalar. The sensor fault E_{Az} is equivalent to a two-dimensional fault F_{Az} [8–10]:

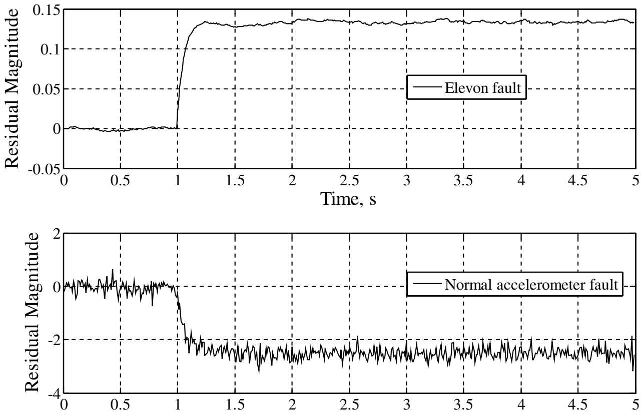

Fig. 1 Elevon fault and normal accelerometer fault (only corresponding to F_{Az}^{2}) isolation residuals.

$$\dot{x}(t) = Ax(t) + F_{Az}m_{Az} \tag{20}$$

where

$$F_{Az} = [F_{Az}^{1}\ \ F_{Az}^{2}] = \begin{bmatrix} 0 & 0.9986 \\ 0 & 0.0534 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

The elevon fault is given simply as $F_{\delta} = B_{\delta}$. Because CF_{δ}, CF_{Az}^{1}, and CF_{Az}^{2} are all nonzero and because none of the triple (C, A, F_{δ}), (C, A, F_{Az}^{1}), and (C, A, F_{Az}^{2}) have invariant zeros, the detection subspaces are $T_1 = \text{span}\{F_{\delta}\}$, $T_2 = \text{span}\{F_{Az}^{1}\}$, and $T_3 = \text{span}\{F_{Az}^{2}\}$. The eigenvalues with respect to T_1, T_2, and T_3 are assigned as (-15), (-10), and (-10), respectively. Set $\Lambda = I$ and $\gamma = 12.5$ (which is about 12.4 in [10]). The computed observer gain L and projection matrix H are

$$L = \begin{bmatrix} 3.5969 & -14.17 & -18.63 & 131.25 \\ -18.501 & -0.89718 & -0.997 & 6.1787 \\ -14.078 & 0.12738 & 0 & 0.73903 \\ -112.54 & -54.008 & 333.44 & 355.03 \\ 0.019313 & -1.8819 & 0 & -0.090947 \end{bmatrix}$$

$$H = \begin{bmatrix} 0.1022 & 0 & 0 & 0 \\ -11.158 & -5.0464 & 33.344 & 35.52 \\ -0.32548 & -2.0506 & 0 & 15.214 \end{bmatrix}$$

Thus, the eigenvalues of $(A + LC)$ are -0.91, -1.66, -10, -10, and -15. The fault scenarios are simulated by adding a 2-deg bias to the elevon deflection after 1 s and adding a 2 ft/s^2 bias to the accelerometer signal after 1 s. Figure 1 shows residual histories in which a zero-mean uncorrelated white noise process with unit spectral density is applied to wind gust and 10% rms noise is added to four sensor measurements. Clearly, both actuator and sensor fault are identified correctly.

Conclusions

In this Note, the fault detection and isolation problem is studied. The concept of detection spaces [10] is used and embedded into the eigenstructure assignment method. In [10], the modified algebraic Riccati equation has no associated Hamiltonian, and so it is difficult to find necessary and sufficient conditions for when a solution exists. In this Note, if the solution for LMI equations exists, it automatically satisfies sufficient and necessary conditions. This idea is new. Also, the computation efficiency is increased by the eigenstructure assignment method and LMI technique.

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