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Time segmented least squares identification of base isolated buildings

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Abstract

In this article two new approaches are presented for time domain identification of base isolated buildings from recorded response during earthquakes: (1) a least squares technique with time segments is developed to identify the piece-wise linear system properties; and (2) an observer is used to estimate the unmeasured states and initial conditions of different time segments. In base isolated buildings changes in dynamic properties occur during earthquake response due to nonlinear behavior. Hence, a multi-input and multi-output technique using time segments is developed for piece-wise linear system identification. The primary advantage of the developed time segmented technique is that it can be applied to windows of time history instead of the entire duration of earthquake response. The developed technique (1) starts with identification using the entire duration of the earthquake response; (2) evaluation of time segments during which the identified response differs significantly from the recorded response to establish windows of time history during which refined identification is necessary; and (3) identification of the change in dynamic properties in the established windows using the observer based time segmented least squares approach. Only partial state measurements are usually available for identification. Hence, an observer is used to estimate the unmeasured states and initial conditions needed for different time segments. By comparing identified response with recorded response, of an actual base isolated building which experienced Northridge earthquake, it is shown that the change in dynamic system parameters, such as periods and damping ratios, due to nonlinear response, are reliably estimated using the presented technique.

Keywords: Time segmented least squares; Dynamic observer; System Identification

1. Introduction

Identification of dynamic properties of buildings has been the subject of study for several decades. Several linear system identification techniques have been developed for this purpose. Beck et al. [1] have developed an efficient method called 'the modal minimization method'. The method is an output error method and uses modes as parameters. The method was found to be efficient for identifying dynamic properties of a 42-story building. Lin and Papageorgiou [2–4] have applied the modal minimization method in several buildings, which experienced strong earthquakes. They studied buildings with closely spaced frequencies that exhibit strong torsional response [2]. They also used the modal minimization method to study soil structure interaction [3]. Papageorgiou and Lin [4] also performed system identification of repaired buildings and found that micro-cracks in structural members dissipate considerable energy, which causes increase in damping. Lin and Mau [5] used the least squares output error method and were able to identify torsional modes due to lateral-torsional excitation. Safak [6] has presented the discrete time domain system identification methods, including recursive methods. Safak [7] has also presented the identifications of linear and nonlinear systems using recursive techniques. Safak and Celebi [17] applied discrete time domain method to identify the properties of a 49-story building. They found the method to be reliable and effective. Shinkozuka and Ghanem and Ghanem and Shinkozuka [8,9] have carried out theoretical and experimental verification of system identification methods, mainly the extended Kalman filter, the maximum likelihood estimation, the recursive least squares, and the recursive instrumental variable with moving window [18]. Marmarelis and Udwadia [10] used the Weiner technique to track the evolution of nonlinearities in a 9-story building. The Weiner technique is a nonparametric identification technique, which is computation intensive. Hence, there is a need to develop efficient techniques for identifying nonlinear structures.

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Two new approaches are proposed in this study to perform system identification of base-isolated buildings. A least squares technique with time segments is developed to identify piece-wise linear system properties of nonlinear base isolated buildings. Full state measurements are usually not available; hence, a reduced order observer is used to estimate the unmeasured states and initial conditions of different time segments.

In base isolated buildings high damping elastomeric bearings or Lead–rubber bearings attached between the superstructure and the foundation provide lateral flexibility and energy dissipation capacity. Flexible behavior of elastomeric bearings leads to longer fundamental period—longer than both its fixed base period and predominant periods of ground motion. The response is reduced due to this period shift in the dominant fundamental mode. Yielding and nonlinear hysteretic behavior of the bearing adds significant energy dissipation to the system. System identification can be used to track nonlinear behavior. In this study piece-wise linear representation of nonlinear systems is adopted for identification. Time segmented technique is proposed for identification of piece-wise linear system properties. The proposed identification technique is used to estimate the change in periods and damping ratios. Comparisons between recorded and identified response of an actual base isolated building, which experienced Northridge earthquake is presented. It is shown that evolving system parameters during the nonlinear response can be reliably estimated using the developed observer based time segmented least squares technique.

2. Time segmented least squares identification

The equation of motion is as follows

$$\bar{\mathbf{M}}\ddot{\mathbf{u}} + \bar{\mathbf{C}}\dot{\mathbf{u}} + \bar{\mathbf{K}}\mathbf{u} = -\bar{\mathbf{M}}\mathbf{R}\ddot{u}_{\mathrm{g}} = -\mathbf{f} \tag{1}$$

where $\ddot{\mathbf{u}}, \dot{\mathbf{u}}, \mathbf{u}$ are the acceleration, velocity, and displacement vectors respectively, $\ddot{u}_{\mathrm{g}}$ is the ground acceleration, $\mathbf{R}$ is a unit column vector, $\bar{\mathbf{M}}$ is the mass matrix, $\bar{\mathbf{C}}$ is the damping matrix, and $\bar{\mathbf{K}}$ is the stiffness matrix.

The state space equation is formulated as follows by defining

$$\mathbf{x}_1 = \mathbf{u}$$

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 = \dot{\mathbf{u}}$$

Thus the equation of motion Eq. (1) can be written as

$$\dot{\mathbf{x}}_2 = -\bar{\mathbf{M}}^{-1}\bar{\mathbf{K}}\mathbf{x}_1 - \bar{\mathbf{M}}^{-1}\bar{\mathbf{C}}\mathbf{x}_2 - \bar{\mathbf{M}}^{-1}\mathbf{f}$$

The state space equation is

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{f} \tag{2}$$

where

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix}.$$

$\mathbf{A}$ and $\mathbf{B}$ are matrices defined as follows

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\bar{\mathbf{M}}^{-1}\bar{\mathbf{K}} & -\bar{\mathbf{M}}^{-1}\bar{\mathbf{C}} \end{bmatrix}; \qquad \mathbf{B} = \begin{bmatrix} 0 \\ -\bar{\mathbf{M}}^{-1} \end{bmatrix}$$

The output equation is formulated as

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{f} \tag{3}$$

where $\mathbf{y}$ is the output vector and $\mathbf{C}$, $\mathbf{D}$ are the parameter matrices defined as follows

$$\mathbf{C} = \mathbf{I}; \qquad \mathbf{D} = \mathbf{0}$$

Eqs. (2) and (3) define a linear time invariant multi-input multi-output dynamic system in state space form; the equations in discrete time are

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{f}(k) \tag{4}$$

and

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{f}(k) \tag{5}$$

Taking z transform of Eqs. (4) and (5)

$$\mathbf{y}(z) = \mathbf{H}(z)\mathbf{f}(z) \tag{6}$$

where

$$\mathbf{H}(z) = \mathbf{C}(z\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}$$

In general form, when $\mathbf{H}(z)$ is parameterized as $\mathbf{B}(z)/\mathbf{A}(z)$ where $\mathbf{B}$ and $\mathbf{A}$ are polynomials in z^{-1}.

$$\mathbf{A}(z) = \mathbf{1} + \mathbf{a}_1 z^{-1} + \mathbf{a}_2 z^{-2} + \cdots + \mathbf{a}_n z^{-n}$$

$$\mathbf{B}(z) = \mathbf{b}_1 z^{-1} + \mathbf{b}_2 z^{-2} + \cdots + \mathbf{b}_m z^{-m}$$

Hence

$$\mathbf{H}(z) = \frac{\mathbf{B}(z)}{\mathbf{A}(z)} \tag{7}$$

$$\mathbf{y}(z) = \frac{\mathbf{B}(z)}{\mathbf{A}(z)}\mathbf{f}(z) \tag{8}$$

which can be written as

$$\mathbf{A}(z)\mathbf{y}(z) = \mathbf{B}(z)\mathbf{f}(z)$$

In discrete time

$$\mathbf{y}(k) = -\mathbf{a}_1\mathbf{y}(k-1)\cdots - \mathbf{a}_n y(k-n) + \mathbf{b}_1\mathbf{f}(k-1)\cdots + \mathbf{b}_m\mathbf{f}(k-m) \tag{9}$$

$$\mathbf{y}(k) = \boldsymbol{\theta}^T\boldsymbol{\Phi}(k)$$

where

$$\boldsymbol{\theta} = (\mathbf{a}_1, \mathbf{a}_2, \ldots \mathbf{a}_n, \mathbf{b}_1, \mathbf{b}_2, \ldots \mathbf{b}_m)^T$$

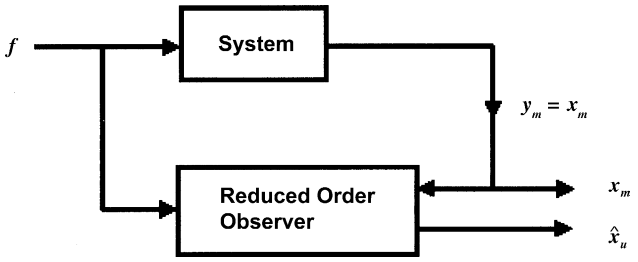

Fig. 1. Block diagram showing reduced order observer.

$$\mathbf{\Phi}(k) = (-\mathbf{y}^T(k-1), -\mathbf{y}^T(k-2), \cdots - \mathbf{y}^T(k-n), \mathbf{f}^T(k-1), \mathbf{f}^T(k-2), \ldots \mathbf{f}^T(k-m))^T$$

Hence identification error becomes

$$\boldsymbol{\varepsilon}(\boldsymbol{\theta}, k) = \mathbf{y}(k) - \boldsymbol{\theta}^T \mathbf{\Phi}(k) \tag{10}$$

Least square criterion can be defined as

$$J_N(\boldsymbol{\theta}, N) = \frac{1}{N} \sum_{k=1}^{N} \|\mathbf{y}(k) - \boldsymbol{\theta}^T \mathbf{\Phi}(k)\|^2 \tag{11}$$

Least square estimate is given by

$$\hat{\boldsymbol{\theta}}_N = \left[\frac{1}{N} \sum_{k=1}^{N} \mathbf{\Phi}(k) \mathbf{\Phi}^T(k) \right]^{-1} \frac{1}{N} \sum_{k=1}^{N} \mathbf{\Phi}(k) \mathbf{y}^T(k) \tag{12}$$

Since output $\mathbf{y}(k)$ does not reflect full state, a reduced order observer is used to estimate the unmeasured states, as shown in Fig. 1. Since the full system satisfies the observability condition, a reduced order observer [11] is developed using

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_{\mathrm{m}} \\ \mathbf{x}_{\mathrm{u}} \end{bmatrix}; \qquad \mathbf{A} = \begin{bmatrix} \mathbf{A}_{\mathrm{mm}} & \mathbf{A}_{\mathrm{mu}} \\ \mathbf{A}_{\mathrm{um}} & \mathbf{A}_{\mathrm{uu}} \end{bmatrix};$$
$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{\mathrm{m}} \\ \mathbf{B}_{\mathrm{u}} \end{bmatrix}; \qquad \mathbf{C} = [\mathbf{I}_{\mathrm{m}} \quad \mathbf{0}] \tag{13}$$

in which $\mathbf{x}_{\mathrm{m}}$ is measured states and $\mathbf{x}_{\mathrm{u}}$ is unmeasured states.

$$\hat{\mathbf{x}}_{\mathrm{m}}(k) = \mathbf{x}_{\mathrm{m}}(k) \tag{14}$$
$$\hat{\mathbf{x}}_{\mathrm{u}}(k+1) = \mathbf{A}_{\mathrm{r}} \hat{\mathbf{x}}_{\mathrm{u}}(k) + \mathbf{K}_{\mathrm{r}} \mathbf{y}_{\mathrm{r}}(k) + \mathbf{P}_{\mathrm{r}}(k)$$

where

$$\mathbf{A}_{\mathrm{r}} = \mathbf{A}_{\mathrm{uu}} - \mathbf{K}_{\mathrm{r}} \mathbf{A}_{\mathrm{mu}}$$

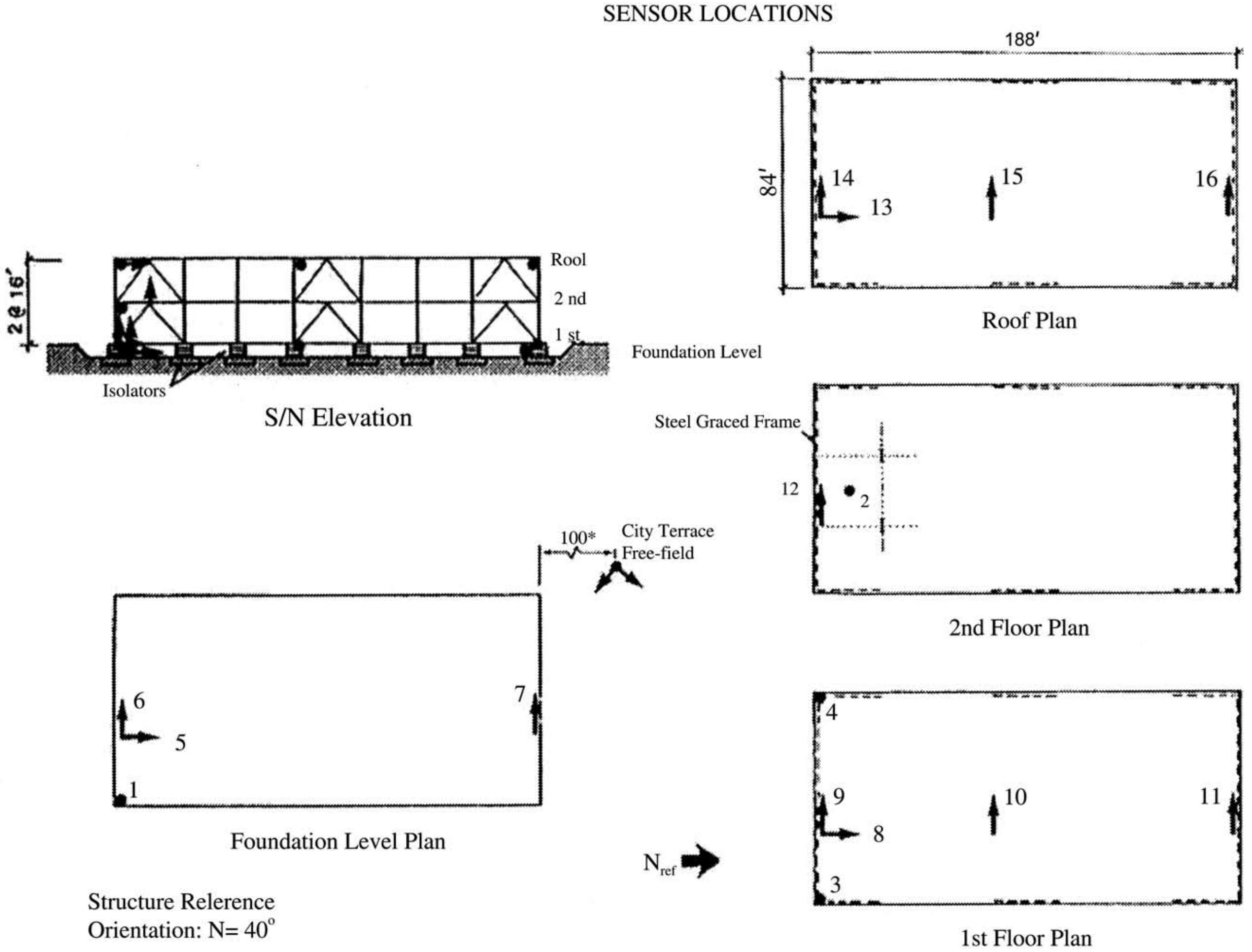

Fig. 2. FCC building elevation and sensor locations.

Table 1
Recorded peak values of response in EW and NS direction (see Fig. 1 for sensor locations)

Channel no.	6	7	9	10
Acceleration (g)	0.22	0.19	0.21	0.23
Channel no.	11	14	15	16
Acceleration (g)	0.35	0.24	0.32	0.25
Channel no.	5	8	13	Free field
Acceleration (g)	0.18	0.07	0.09	0.32

$$\mathbf{y}_r(k) = \mathbf{x}_m(k+1) - \mathbf{A}_{mm}\mathbf{x}_m(k) - \mathbf{B}_m\mathbf{f}(k)$$

$$\mathbf{P}_r(k) = \mathbf{A}_{um}\mathbf{x}_m(k) + \mathbf{B}_u\mathbf{f}(k)$$

$\mathbf{K}_r$, reduced order observer gain matrix.
In discrete time

$$\hat{x}_u(k+1) = \mathbf{A}_r\hat{x}_u(k) + \mathbf{K}_r\mathbf{y}_r(k) + \mathbf{P}_r(k) \quad (15)$$

The identification of nonlinear system is approximated by piece-wise linear at systems. Eqs. (1)–(15) are applied for each piece-wise system. It is assumed that each system

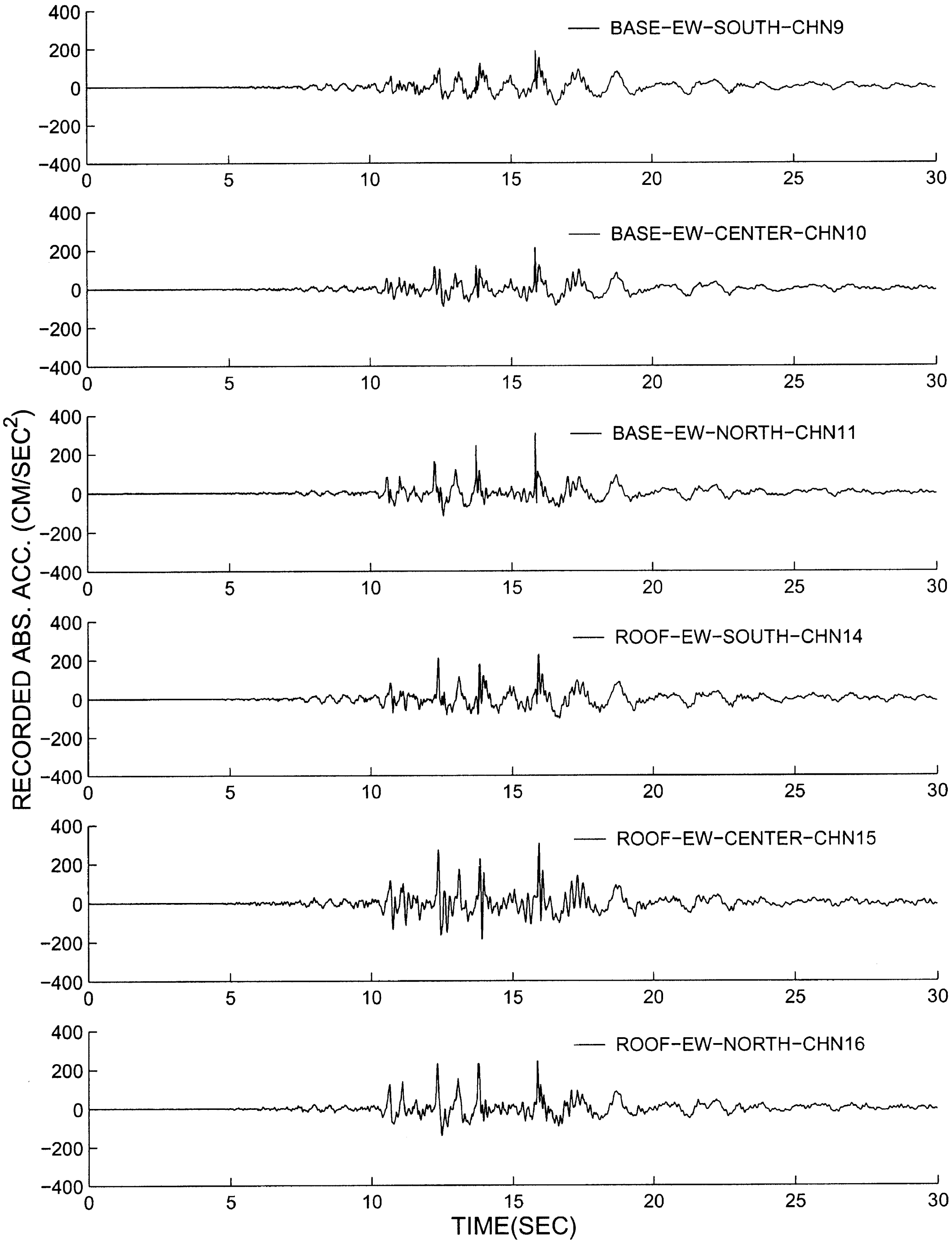

Fig. 3. Recorded acceleration time histories of FCC building in Northridge earthquake.

Table 2
Identified and computed periods and damping ratios of base Isolated FCC building in EW direction

Mode	Computed		Identified (0–30 s)		Identified (12–16 s)		Identified (16–30 s)	
(1)	T (s) (2)	ξ (%) (3)	T (s) (4)	ξ (%) (5)	T (s) (6)	ξ (%) (7)	T (s) (8)	ξ (%) (9)
1	1.38	14	1.21	10	0.94	8	1.35	11
2	0.21	12	0.2	10.1	0.18	10	0.18	10
3	0.12	5	0.12	5	0.11	5	0.11	5

maintains constant equivalent linear stiffness and damping properties. The developed technique (1) starts with identification using the entire duration of the earthquake response; (2) evaluation of time segments during which the identified response differs significantly from the recorded response to establish windows of time history during which refined identification is necessary; and (3) identification of the change in dynamic properties in the established windows using the observer based time segmented least squares approach.

3. Numerical example

For the numerical example the Fire Command and Control (FCC) base isolated building [17] is chosen, since, significant change in dynamic properties occurred during the nonlinear response of this building [12] in the Northridge earthquake., which is a good test case for the presented time segmented least squares technique. FCC is a two-story base isolated building, 57.3 m (188 ft) long and 25.6 m (84 ft) wide, with seven bays in the North–South (NS) direction and three bays in the East–West (EW) direction. Fig. 2 shows the plan and elevation of the building. The isolation system is made up of 32 high damping rubber bearings. The building has been instrumented by CSMIP [13]; the sensor locations are shown in Fig. 2. The sensors (accelerometers) are located at the foundation, first floor, second floor, and roof in the North–South (NS) and East–West (EW) direction. The peak values of the recorded acceleration, in the EW and NS direction, during Northridge earthquake are shown in Table 1. The acceleration time histories are presented in Fig. 3.

The presented acceleration time histories of Channel Number [CHN] 9, 10, 11 at the base level and of CHN 14, 15, 16 at the roof level, in the EW direction, shown in Fig. 3,

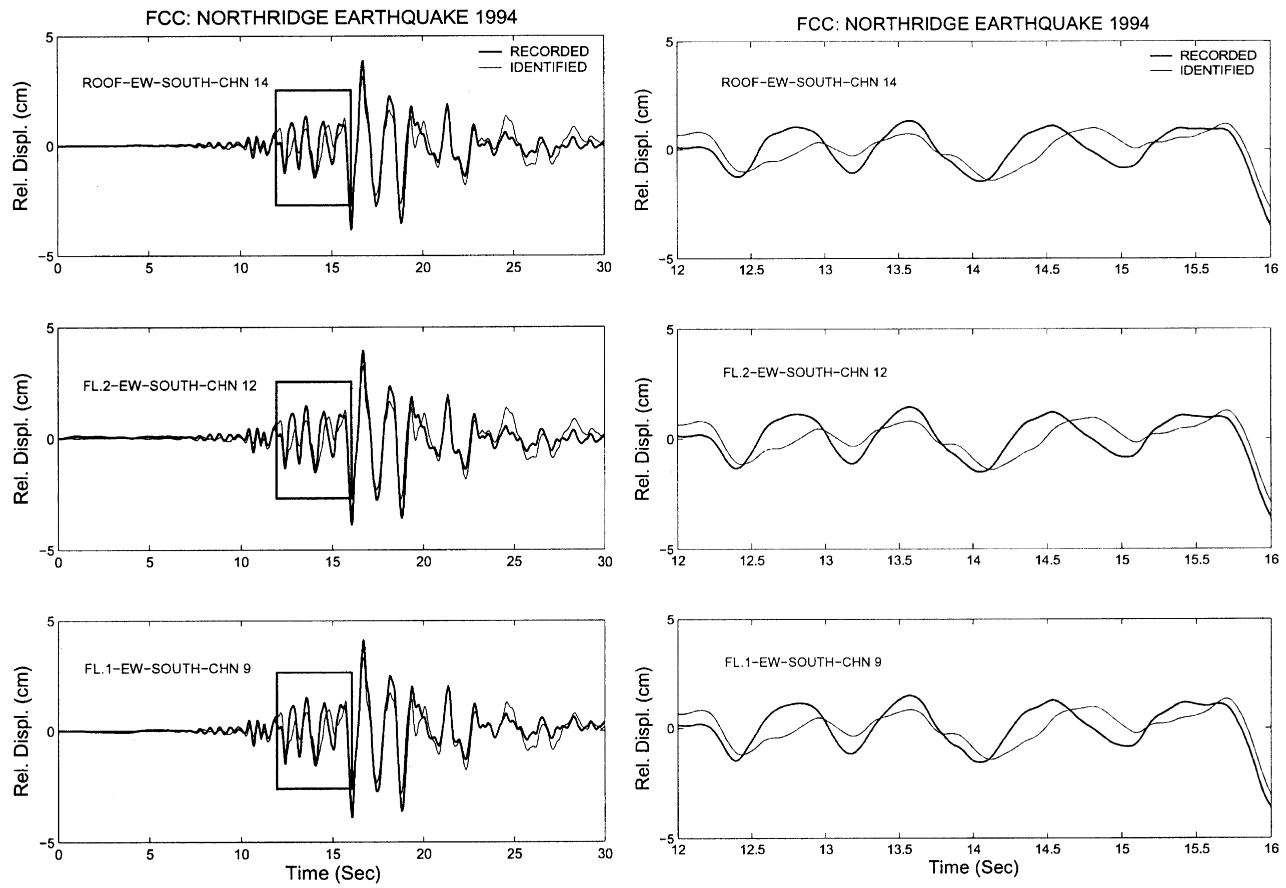

Fig. 4. Comparison of recorded and simulated displacement response obtained from 0 to 30 s Identification (a) time history response from 0 to 30 s; (b) 12–16 s response time history magnified to show the differences in identified and recorded responses.

exhibit sharply increased higher mode response, in each upper half cycle, between 12 and 16 s time interval, increasing the acceleration from 0.22 g at the foundation level to 0.35 g at the roof level. Nagarajaiah and Sun [12] have shown that increased higher mode response was caused due to one-sided impact. The impact occurred against concrete entry bridge in the northeast corner of the building [14]. Nagarajaiah and Sun [12] have also shown that impact occurred only in the EW direction and no impact occurred in the NS direction. Hence, only identification in the EW direction is considered. CHN 9, 10, 12, 14 and 15 are used for identification. For CHN 10 and 15 the unmeasured second-floor response is estimated using the reduced order observer. For CHN 9, 12, and 14 the observer is used for estimation of initial conditions needed for different time segments of identification.

A two-dimensional (2D) analytical model is developed in the EW direction based on a linear superstructure and equivalent linear isolation system, with one degree of freedom (DOF) per floor. The building superstructure details including beam, column, bracing details from building drawings and bearing prototype test results [15] are used for developing the 2D-analytical model. Superstructure fixed base properties with $T_1 = 0.37$ s, $T_2 = 0.13$ s, with damping ratio of 5% in both modes, equivalent linear isolation stiffness of $K_b = 12.25$ kN/cm, and isolation damping of 16% are used to develop the model. The computed periods and damping ratios in the EW direction are shown in Table 2.

The identification is performed in the EW direction from 0 to 30 s, using the least squares technique to establish the average dynamic properties of the system during Northridge earthquake. Identified periods and damping ratios are shown in Table 2 (columns 4 and 5). The identified state space model is used to simulate the response from 0 to 30 s. The simulated and recorded displacements and velocities are compared in Figs. 4(a) and 5(a).

Nagarajaiah and Sun [12] have shown that significant changes in dynamic properties occurred in FCC building due to impact between 12 and 16 s. In cases in which such prior information is not available, evaluation of time segments during which the identified response differs significantly from the recorded response can be used to establish windows of time history during which refined identification is necessary. The primary advantage of the developed time segmented technique is that it can be applied

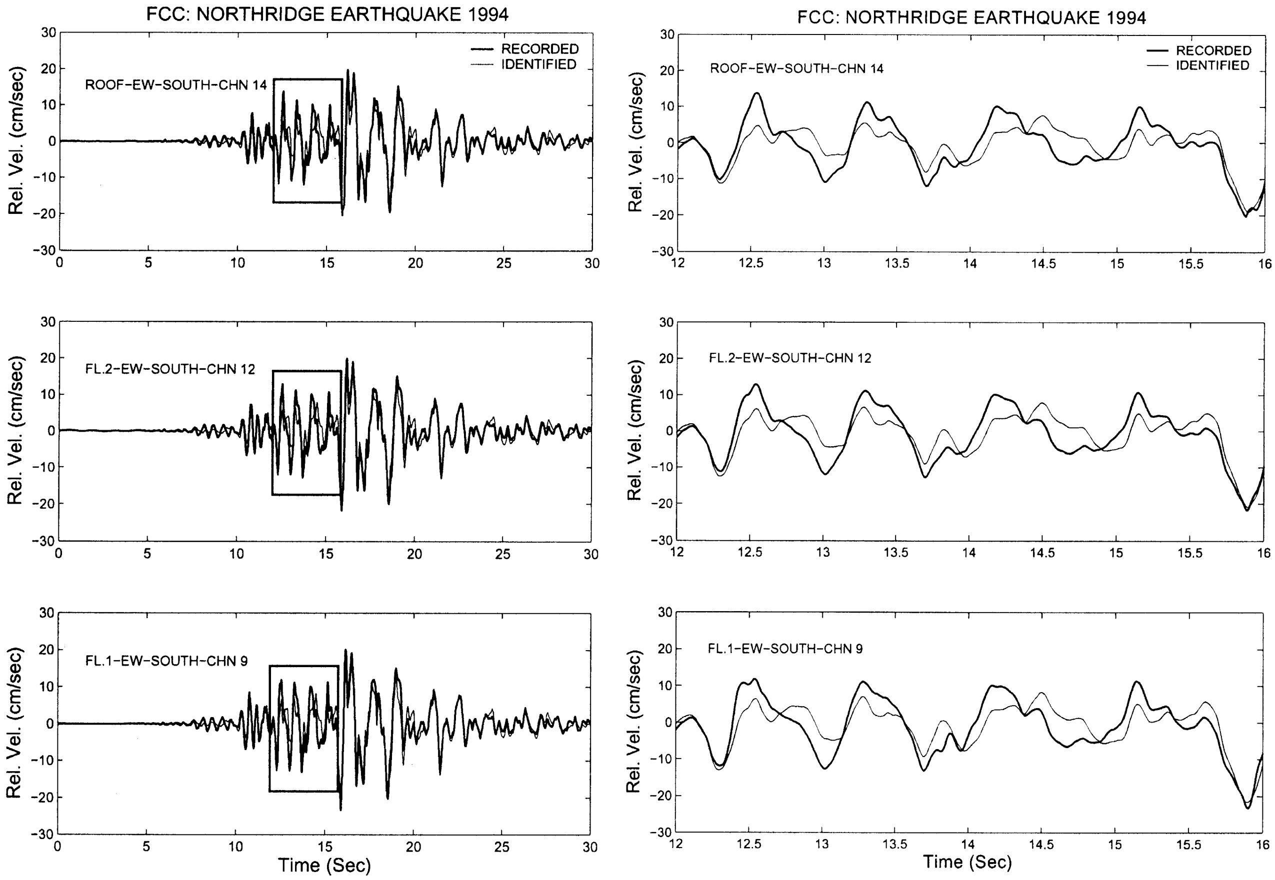

Fig. 5. Comparison of recorded and simulated velocity response obtained from 0 to 30 s identification (a) time history response from 0 to 30 s; (b) 12–16 s response time history magnified to show the differences in identified and recorded responses.

to windows of time history instead of the entire duration of earthquake response. Although, engineering judgment needs to be exercised in establishing time segments for refined identification.

Close examination of the time history response between 12 and 16 s indicates that the comparison between recorded and identified response does not correlate well as shown in Figs. 4(b) and 5(b). In order to evaluate the ability of the proposed time segmented least squares identification technique, a time segment between 12 and 16 s is chosen for refined identification. The initial conditions needed for identification are estimated using the observer. The periods and damping ratios obtained from identification performed from 12 to 16 s are shown in Table 2 (columns 6 and 7). It is clearly seen from the results that the fundamental period is significantly reduced to 0.94 s, primarily due to impact and contact of the entry bridge; the bridge acts as a stiff spring. The second and third modes are not affected significantly by the impact. The updated state space model between 12 and 16 s is used to simulate the response. The simulated and recorded displacements and velocities are shown in Fig. 6. The gap for one-sided impact is shown in Fig. 6(a). The upper half cycles of displacement response are limited by the gap, shown in Fig. 6(a), due to impact; whereas, the lower half cycles of displacement response are complete. Such a response would not occur if it was because of the nonlinearity in the bearings—the nonlinear response would be similar in upper and lower half response cycles. Although, in general it would be difficult to identify the specific nonlinearity that causes the change in dynamic properties.

The comparison between the simulated and recorded responses between 12 and 16 s, shown in Fig. 6(a) and (b), is significantly improved because of the time segmented least squares identification allowing the algorithm to converge more precisely to the properties that exist during this period. This demonstrates the efficiency of the time segmented least square technique developed and presented in this article.

Next, in order to identify the dynamic properties after 16 s and to examine the true isolation periods that existed during Northridge earthquake a time segment of 16–30 s is chosen. Identified periods and damping ratios are shown in Table 2 (columns 8 and 9); the agreement between the computed (columns 2 and 3) and identified periods and damping ratios (columns 8 and 9) is satisfactory.

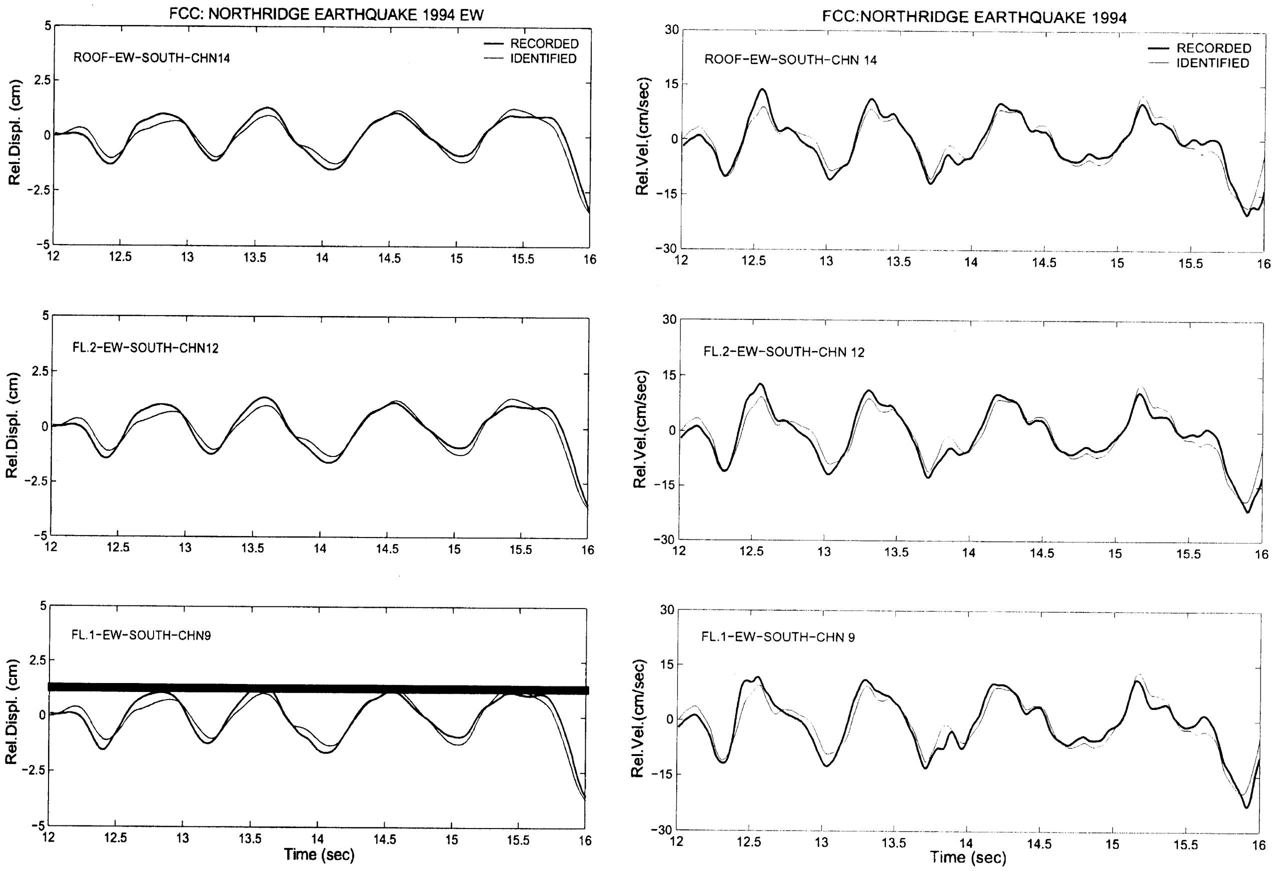

Fig. 6. Comparison of recorded and simulated time history response obtained from 12 to 16 s. Time segment identification (a) displacement response (thick line shows the gap for one sided impact); (b) velocity response.

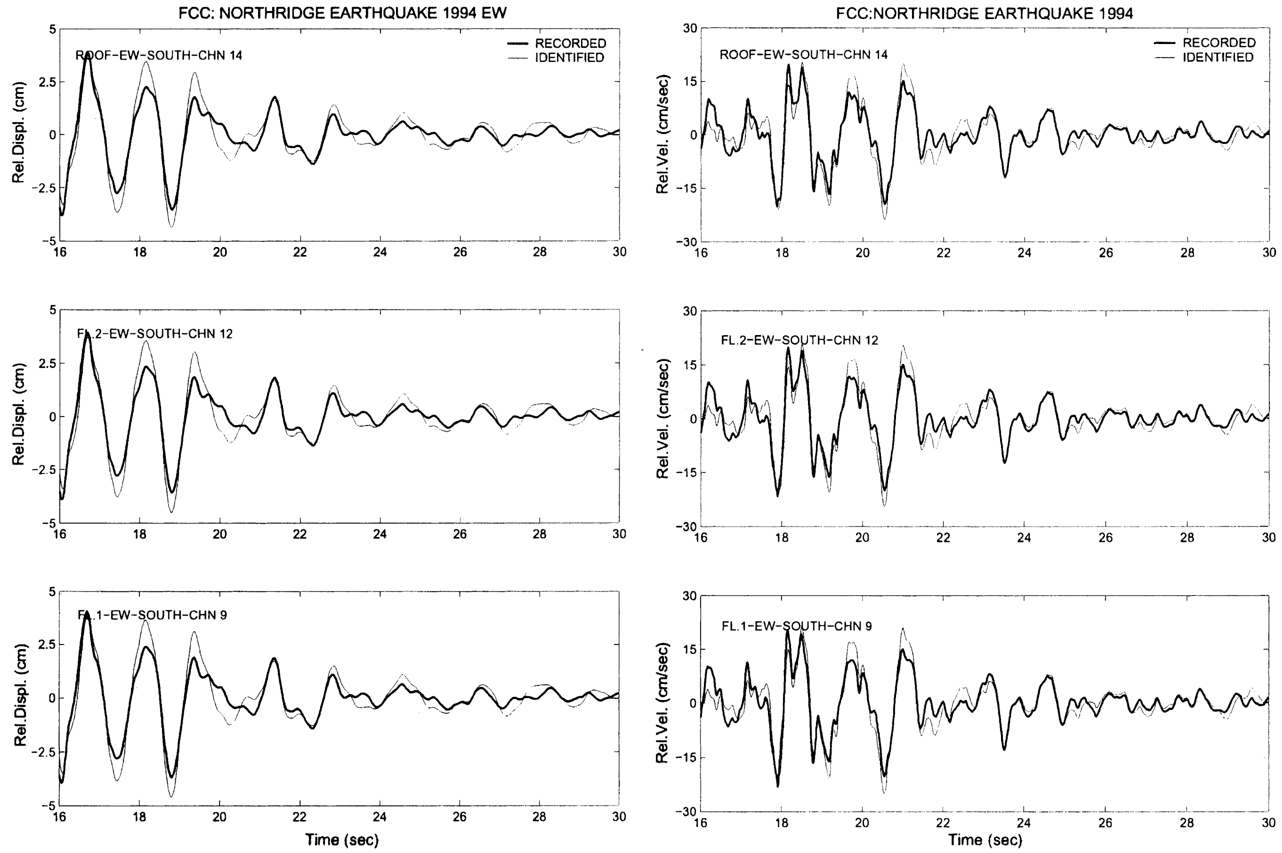

Fig. 7. Comparison of recorded and simulated time history response obtained from 16 to 30 s time segment identification (a) displacement response; (b) velocity response.

The updated state space model identified is used to simulate the response from 16 to 30 s. The simulated and recorded displacements and velocities are compared in Fig. 7. The comparison is satisfactory; although, two windows one between 16 and 19.5 s and the other between 19.5 and 30 s would lead to better correlation.

The three mode shapes identified during time segment (1) 12–16 s are shown in Fig. 8, and (2) 16–30 s are shown in Fig. 9. In the first mode, during 16–30 s shown in Fig. 9, most of the deformation occurs in the bearings within negligible deformation in the superstructure—a typical feature of the fundamental mode of base isolated structures in which the superstructure behaves almost as a rigid body [16]; however, in the first mode, during 12 to 16 s shown in Fig. 8, deformation occurs in both the bearings and the superstructure, due to impact. The effect of impact can also be clearly seen in the transfer functions during and after impact shown in Fig. 10. During impact the frequency is nearly 1.2 Hz or 0.84 s and after impact the frequency is nearly 0.8 Hz or 1.25 s. Note both 0.84 and 1.25 s are approximate periods obtained from frequency domain represent average properties. The periods shown in Table 2 from time segment identification are more accurate.

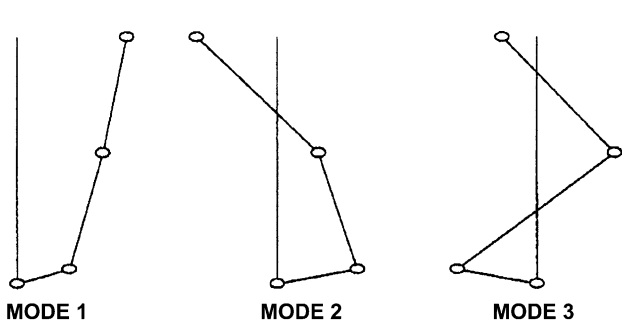

Fig. 8. Modal response in EW direction during impact.

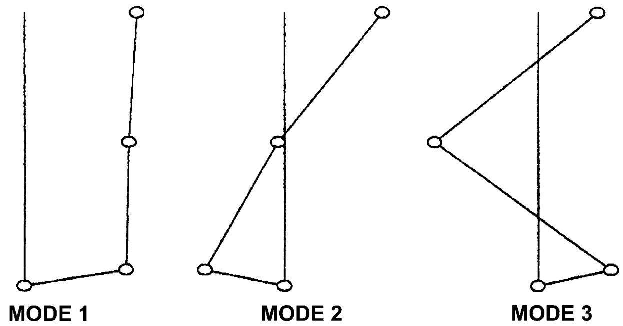

Fig. 9. Modal response in EW direction after impact.

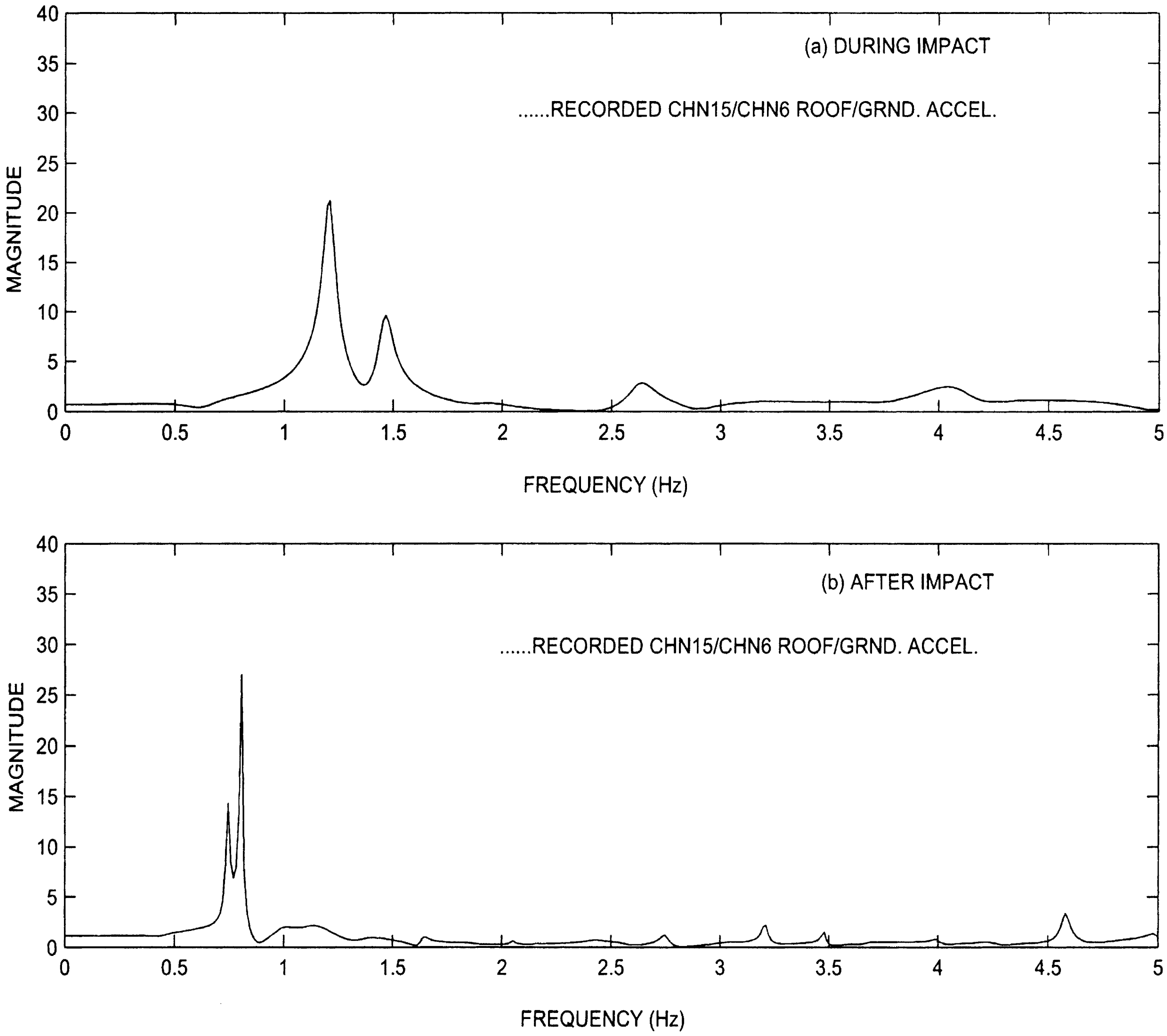

Fig. 10. Recorded transfer function in EW direction: (a) base isolation with impact; (b) base isolation without impact.

4. Conclusions

Based on the results presented in this article it can be concluded that the presented multi-input multi-output system identification technique can be used to track piece-wise linear representation of nonlinear behavior of base isolated buildings. Time segmented least squares observer based technique, proposed for identification of piece-wise linear system properties, is found to be effective in estimating significant changes in periods and damping ratios during specific time windows. Although, the primary advantage of the developed time segmented technique is that it can be applied to windows of time history instead of the entire duration of earthquake response, engineering judgment is needed in establishing the time windows for refined identification. The presented comparisons between recorded and simulated response of an actual base isolated building which experienced Northridge earthquake show that evolving system parameters can be reliably estimated using the developed technique.

Acknowledgements

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