Discussion of "Behavior and Modeling of Nonprismatic Members Having T-Sections" by Can Balkaya

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The writer presents a parametric study devoted to investigate the behavior of nonprismatic members having T-sections by using three-dimensional (3D) finite-element models (FEMs). Fixed-end moments and stiffness coefficients were computed from the 3DFEMs considering thrust effects. As a result of the parametric study, the writer proposed a practical modeling technique based on two-noded beam elements with an effective length concept to use in frame analysis. In the model proposed by the writer, the effective length of the two-dimensional (2D) beam element representing the haunch evolves as 75% of its actual length.

The writer is well known for his previous studies on haunched beams using elastic finite elements and is to be complimented for the interesting study he presented. In particular, his discussion on the general behavior of nonprismatic members is very insightful for most readers, as he notes that the discontinuity in the location of the centroidal axis on nonprismatic members favors a nonlinear stress distribution over the cross section, and coupling attributable to the arching action. The writer also notes that, for T-sections, the shifting in the location of the centroidal axis produces a relatively small arch depth that results in smaller axial forces than those of rectangular sections. Although the study presented by the writer is very interesting, the discusser wants to do the following observations.

In the Introduction, the writer states that: "Nonprismatic beam members are used commonly in bridge structures and less frequently in building structures." The discusser is of the opinion that this statement is not a rule of thumb and may vary from region to region. For example, in Mexico City RC haunched beams are more commonly used for buildings than for bridges as shown in Fig. 1. In fact, the use of RC haunched beams in buildings has a long tradition in Mexico City (i.e., Tena-Colunga 1994).

In the Introduction, the writer also states that: "Most computer programs do not allow the user to enter the stiffness coefficients." This is a somewhat incorrect and misleading statement, as noted by the discusser in a previous discussion with the writer (Balkaya and Citipitiouglu 1997). Well-known programs like the old TABS and SUPERETABS, ANSR, PC-ANSR, and the DRAIN family of programs, including the more recent versions (DRAIN-2DX and DRAIN-3DX) allow one to include specific stiffness coefficients. Recently, leading commercial software like ETABS (version 6 or higher) allow the user to approximately model tapered elements by defining up to three different segments along the member length with different specific cross section properties. ETABS version 6 considers the following general rules of variation for the cross section properties between segments: constant, linear, parabolic, and cubic.

The writer also states in the Introduction "Tena-Colunga (1996) gave the stiffness formulation for nonprismatic beam elements having T-sections. However, he did not consider the coupling between moment, shear, and axial forces due to arching

Fig. 1. Building with RC haunched beams currently under construction in Mexico City

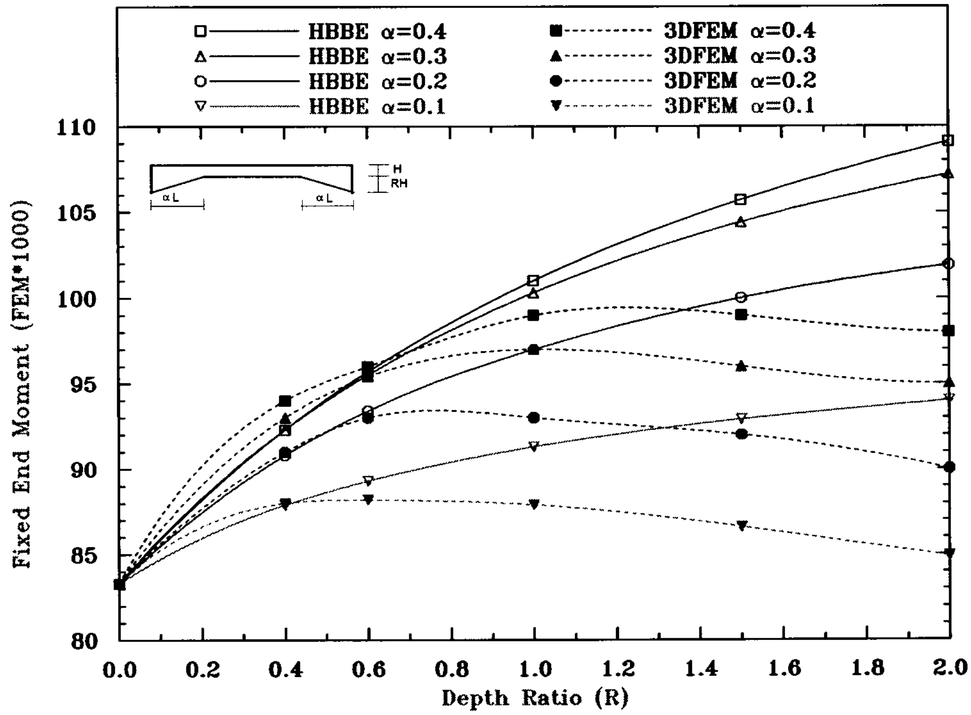

Fig. 2. Comparison of fixed-end moments for case 15

action in the general behavior of nonprismatic members (Balkaya and Citipitiouglu 1997).'' The discusser wants to point out that the results presented in Tena-Colunga (1996) for haunched beams of T- and I-cross sections, as well as the closed-form solutions for linearly tapered elements of rectangular, square, and circular cross sections, the variation of the centroidal axis within the length was taken into account in the formulation, as it could be observed in the expressions used for the T-section shown in the Appendix. These equations were not previously included in Tena-Colunga (1996) for space constraints. Therefore, the influence on the change of the centroidal axis along the length of the haunched elements is being taken into account in the stiffness coefficients and fixed-end moments reported in Tena-Colunga (1996), so the arching action is not completely ignored in such modeling. Perhaps the writer was misled by the discusser in the closure of the discussion done by Balkaya and Citipitiouglu (1997), and then this discusser is to blame for that.

In the section entitled ''Fixed-end Moment and Stiffness Factors,'' the writer states that: ''because there are no available data for the comparison of these results on the T-section effects, fixed-end moment and stiffness factors are checked relatively to those obtained for finite-element and PCA models of nonprismatic members of rectangular sections $H/L=0.10$ ... .'' Also, in the conclusions, the writer starts this section writing ''design tables are currently not available for nonprismatic members having T-sections.'' The discusser calls the attention of the writer, as in Tena-Colunga (1996): (a) Stiffness coefficients and fixed-end moments for T-sections are provided (Figs. 3–6) and, (b) it is mentioned that new design aids for linearly varying haunched beams of T- and I-cross sections are available in Tena-Colunga and Zaldo-García (1994). In fact, Fig. 5 of Tena-Colunga (1996) depicts how a typical design table for haunched T-element available in Tena-Colunga and Zaldo-García (1994) looks like. Also, the unsymmetrical haunched beam presented by the author in Fig. 11(a) has the same geometry that the one presented by Tena-Colunga (1996) in Fig. 2(b); the only difference is that in Tena-Colunga (1996) results are presented for this beam in Figs. 4(b), 5, and 6, and the writer just does a brief discussion on the unsymmetrical arch formation with the help of Fig. 11(b). As Balkaya and Citipitiouglu (1997) have previously discussed the material presented in Tena-Colunga (1996), and then the writer (Balkaya) is familiar with that material, the discusser believes that perhaps the writer decided not to compare his study with the results presented in Tena-Colunga (1996) for whatever reason, and he is probably entitled to do that. Nevertheless, it is unfair with the work of the discusser to state that currently there are not design tables avail-able for nonprismatic members having T-sections, when the writer is aware that design aids are available in a research report of the discusser published in Mexico since 1994.

The discusser considers that, in benefit of the audience of ASCE's *Journal of Structural Engineering*, it would be interesting to compare the results presented by the writer with respect to the ones that can be obtained using the Bernoulli–Euler beam theory formulation for nonprismatic elements presented in Tena-Colunga (1996). Therefore, the results presented by the writer for case 15 (Table 1) of the symmetric haunched beam depicted in Fig. 4 under his 3D FEM modeling (Table 2, Figs. 5, 7, and 8) are compared with those obtained with the formulation presented in Tena-Colunga (1996) and identified as haunched beam, Bernoulli–Euler (HBBE) in Fig. 2 (fixed-end moments), Fig. 3 (stiffness coefficient), and Fig. 4 (carryover factor). It is worth noting that the beam aspect ratio is $L/H=12.5$ taking as a reference the smaller cross section, so these T-beams can be considered well-proportioned beams, and shear deformations may have a moderate impact on the stiffness coefficients.

Fixed-end moments are compared in Fig. 2, and one can do the following observations: (1) under the HBBE and 3D FEM modelings, fixed-end moments increase as the relative length ratio (α) of the haunch increase, so in this regard both models agree, (2) under the HBBE modeling, fixed-end moments increase as

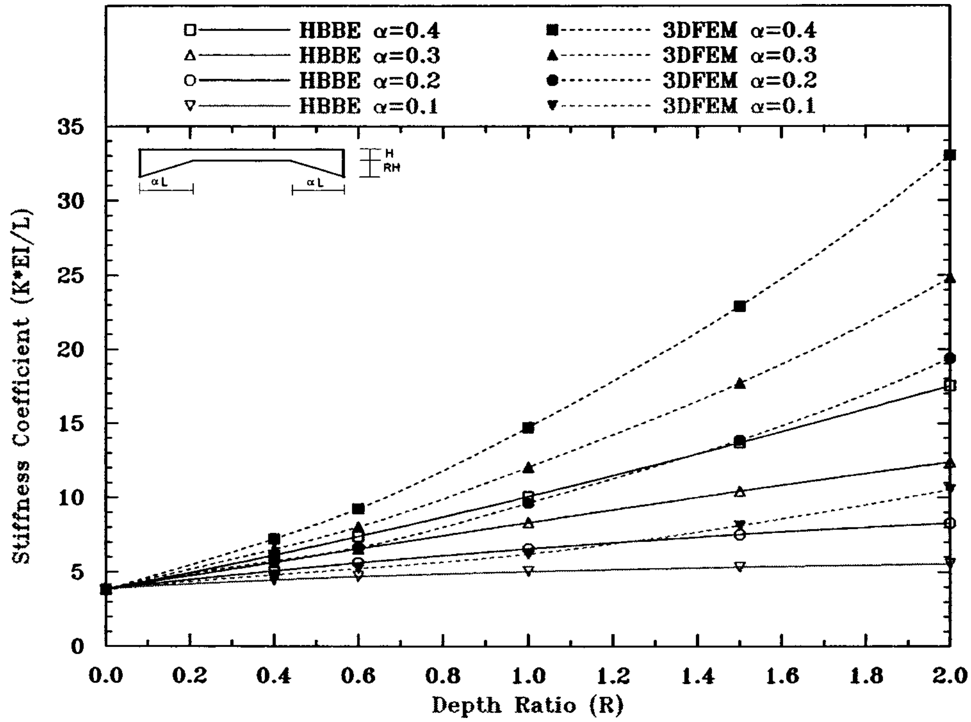

Fig. 3. Comparison of the stiffness coefficient for case 15

haunched depths are increased; however, under the 3DFEM modeling fixed-end moments only increase for depth ratios $R \leqslant 1.0$ and then start to decrease for higher depth ratios. This difference might be attributed to the arching action (by the writer), as in theory; the arching action becomes more important as the haunched depth increases. Nevertheless, the discusser would like the writer to explain further to the ASCE audience how haunched beams with $R=2.0$ are subjected to smaller fixed-end moments than haunched beams with $R=1.0$ for a given α ratio, being haunched beams with $R=2.0$ stiffer than those with $R=1.0$, when common sense (and traditional beam theory) would suggest the opposite? In any case, both methods roughly agree for depth ratios $R \leqslant 0.6$.

Stiffness coefficients [named r_{11x} in Tena-Colunga (1996), $r_{11x}=4EI/L$ for prismatic beams where shear deformations are neglected] are compared in Fig. 3. The stiffness coefficient r_{11x}

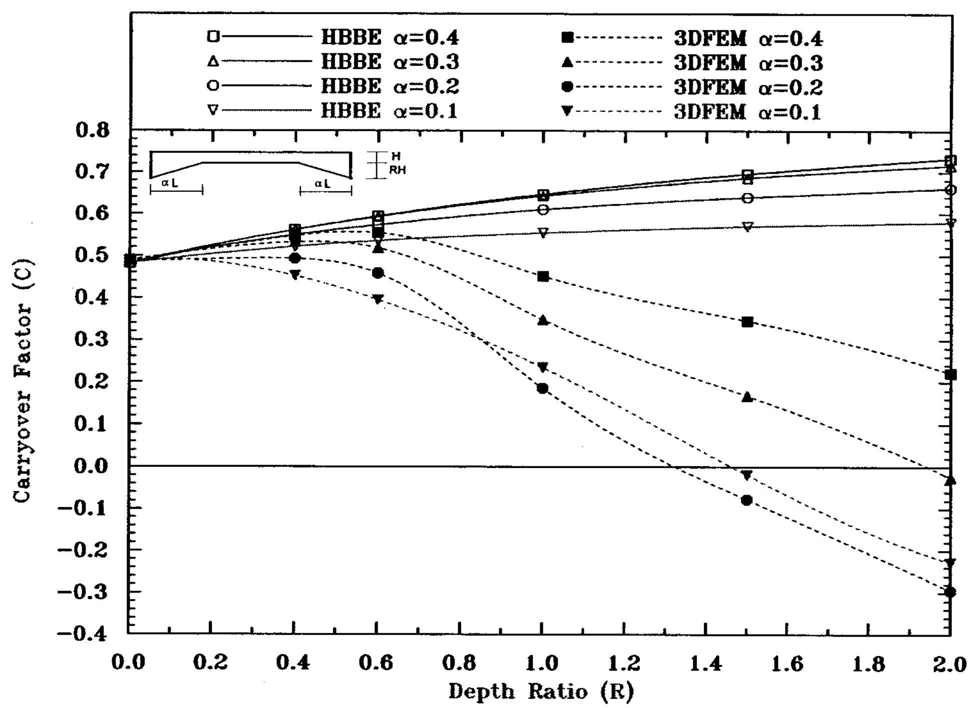

Fig. 4. Comparison of the carryover factor for case 15

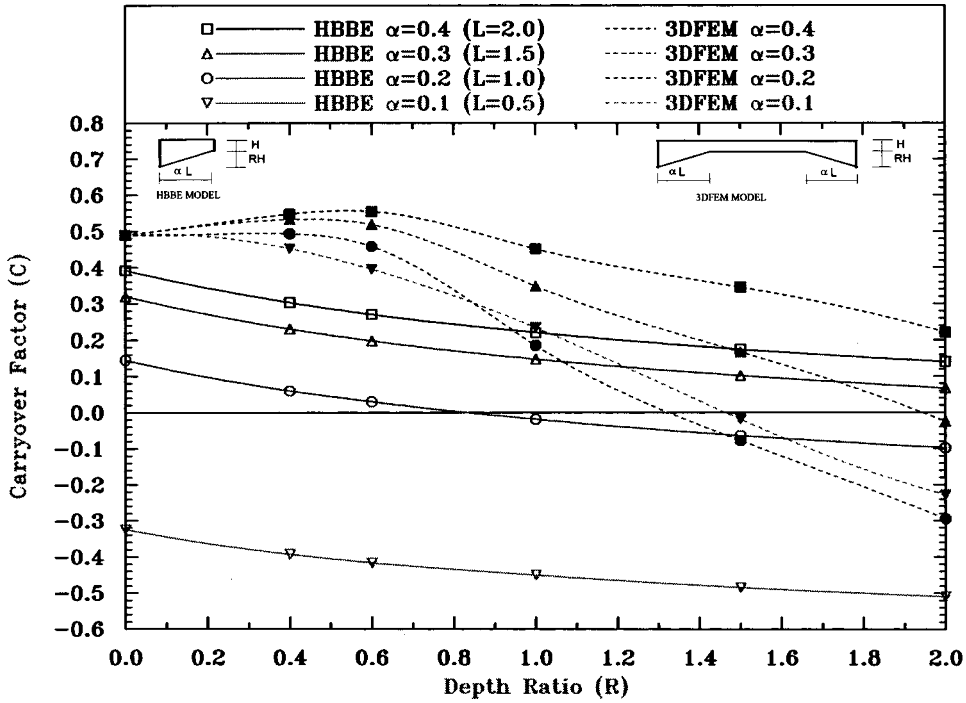

Fig. 5. Comparison of the carryover factor for case 15, considering tapered beams for HBBE models

increases as both the relative length (α) and depth (R) ratios increase for both modelings; however, this stiffness coefficient is much higher for beams with haunch depths $R>0.6$ for a given relative length ratio α under the 3DFEM modeling. If arching action is responsible for this phenomenon then, how can it be explained that fixed-end moments for deeper haunches under the 3D FEM decrease as haunched depths are increased and are smaller than those computed from the HBBE modeling, if r_{11x} increases more rapidly under the 3DFEM as the depth of the haunch increases? In the humble opinion of the discusser, it seems to be an inconsistency here in the results presented by the writer.

Carryover factors (cof) are compared in Fig. 4. For the HBBE models, the carryover factors were computed, following the notation used in Tena-Colunga (1996), as follows:

$$\text{cof}=\frac{r_{12x}}{r_{11x}} \tag{1}$$

It can be observed from Fig. 4 that there is a notorious discrepancy in the computation of the carryover factors between the HBBE and 3DFEM modeling for depth ratios $R>0.6$, as under the HBBE modeling the carryover factors always increase as both α and R increase, whereas under the 3DFEM carryover factors increase and then abruptly decrease as the haunched depths are increased. In addition, negative carryover factors are obtained for $R\geqslant 1.5$ under the 3DFEM for values of $\alpha\leqslant 0.2$. The discusser has the opinion that these results are erroneous and are probably due to the procedure used by the writer to compute carryover factors from finite elements, for the following reasons. First, the results presented by the writer are supposed to be for symmetric haunched beams with aspect ratio $L/H=12.5$ (Fig. 4), so the carryover factors presented by the writer must consider the whole geometry of the haunched beam depicted in Fig. 4. If that is true, then one cannot accept decreasing and even negative carryover factors on beams that are well proportioned, as shear deformations are not very important for such beams. Negative carryover factors can be obtained for very short beams (for example, $L/H<2$) or walls systems, where shear deformations are important. This can be illustrated in a simple way for a general prismatic beam. It can be demonstrated that for a prismatic beam with shear deformations, the carryover factor can be computed as

$$\text{cof}=\frac{r_{12x}}{r_{11x}}=\frac{\dfrac{(2-\Phi_y)EI_x}{L(1+\Phi_Y)}}{\dfrac{(4+\Phi_y)EI_x}{L(1+\Phi_Y)}}=\frac{2-\Phi_y}{4+\Phi_y} \tag{2}$$

where

$$\Phi_y=\frac{12EI_x}{GA_{cy}L^2}=24(1+\nu)\frac{A}{A_{cy}}\left(\frac{r_x}{L}\right)^2 \tag{3}$$

Therefore, from Eq. (2) it is clear that the only way to have a negative carryover factor is if $\Phi_y>2$, and from Eq. (3) it is clear that Φ_y increases as the radius of gyration to length ratio (r_x/L) for the member increases, this is, as the cross-section dimensions for the member get closer to the length of the member, as it is well known. The larger haunch depth considered by the author is $R=2$, so the smaller aspect ratio for the considered haunched beams would be $L/(H+RH)=4.17$ if the section would remain constant (that it is not the case). In any event, the haunched beams with $\alpha=0.4$ would be the closer ones to be considered as short beams that those with $\alpha=0.1$; however, the negative carryover factors are obtained for the last ones ($\alpha=0.1$). How can this inconsistency then be explained?

The discusser ignores the details of how the writer computed the carryover factors using 3D finite elements, but his intuition led him to believe that perhaps the writer obtained the carryover factors (and perhaps the stiffness factors as well) primarily related to the tapered part of the haunched beams, instead of the whole symmetric haunched beams, as it was done under the HBBE modeling. This is illustrated with the help of Fig. 5, where under the HBBE modeling, carryover factors were computed just for the

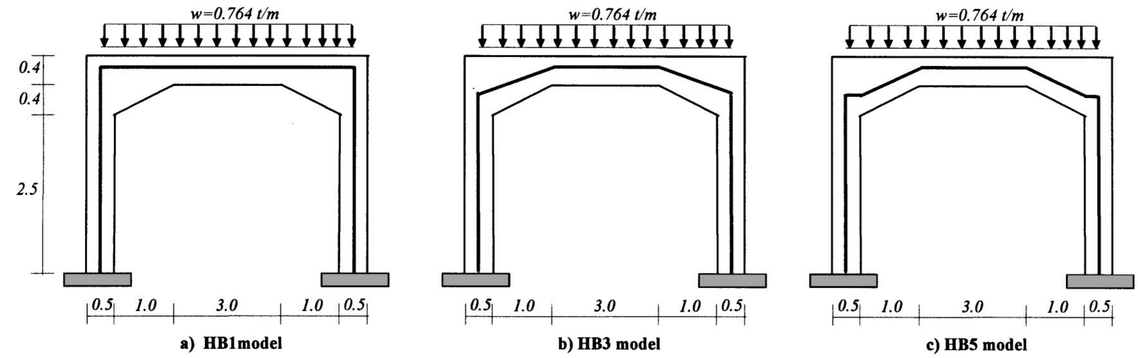

Fig. 6. HB1, HB3, and HB5 models for the frame example of Fig. 13

tapered section of the haunch beams of reference (Fig. 4). It can be observed that if only the tapered part is modeled, carryover factors for $\alpha \leqslant 0.2$ are indeed negative, as the aspect ratio for the tapered part of the beam with respect to the cross section is indeed short. However, the shapes of these curves are different from those presented by the writer, as also shown in Fig. 5.

Finally, this discussion will be somewhat incomplete if the discusser would not compare the result of the frame example presented by the author using 3D FEM (Fig. 13, Table 3) with those using 2D planar frame analysis using the DRAIN-2DX software (Prakash et al. 1992) and modeling the haunched T beams (stiffness coefficients, fixed-end moments) according to the beam theory presented in Tena-Colunga (1996). Three different models were used to represent the haunched beams as depicted in Fig. 6: (1) modeling the haunched beam with a single element (model HB1); (2) modeling the haunched beam with three elements, one for the prismatic mid-section and two to independently model each tapered part (model HB3); and (3) modeling the haunched beam with five elements, using two additional prismatic elements to model the end zones (model HB5). According to what is stated by the writer, the worst modeling should be HB1 model, as a straight beam modeling cannot represent exactly the arching action (except that the variation of the centroidal axis within the length is taken into account in the computation of the stiffness coefficients and fixed-end moments). Model HB3 can represent the arching action reasonably well, but may have problems representing the end zones, as most models based on beam–column elements do. Model HB5 would be the closer one to represent the end zones. It is uncertain and not easy to decide how much of the beam–column joint has to be considered as a rigid end zone in haunched beams for analyses purposes (as it is for prismatic beams too). On this regard, the 3DFEM model used by the writer constitutes a better modeling, but as he recognizes, it is not affordable for many applications, particularly multistory buildings. In order to gain some insight on this subject, the discusser considered three different rigid end-zone assumptions for each model: 100% effective, 50% effective, and 0% effective (no rigid-end zones). A common rule of thumb for medium to high-rise reinforced concrete framed buildings is to consider a 50% effective rigid end zone.

Beam and column design forces computed for the frame example (Fig. 13) under the HB1, HB3, and HB5 modelings (Fig. 6) are compared to those reported by the writer for the 3DFEM in Table 1. It can be concluded that correlations with the 3DFEM model are very reasonable for all models when a 50% effective rigid-end zone is considered. Surprisingly as it may seem, the closer approximation to the 3DFEM model is obtained for the HB1 50% modeling. However, for design purposes it would be more conservative to overestimate design forces at the haunched beam, so in this regard perhaps the HB3 50% model is more appropriate. In any event, this comparative study suggests that: (1) it might be good enough to use up to three suitable beam elements to model frames with haunched beams subjected to gravity loading to obtain reasonable estimates of design forces,

Table 1. Beam and Column Design Forces

		BEAM DESIGN FORCES				COLUMN DESIGN FORCES			
		At Face of Haunch			At Midspan	At Top of Column			At Base
Model	Rigid end zone	Moment (t-m)	Shear (t)	Axial (t)	Moment (t-m)	Moment (t-m)	Shear (t)	Axial (t)	Moment (t-m)
3DFEM		1.220	1.914	0.914	1.016	1.372	0.914	−2.10	−0.913
	100%	1.381	1.754	1.188	0.870	1.422	0.917	−2.10	−0.870
HB5	50%	1.255	1.761	1.141	1.003	1.310	0.870	−2.10	−0.865
	0%	1.116	1.771	1.078	1.152	1.192	0.805	−2.10	−0.820
	100%	1.466	1.937	1.257	0.886	1.486	0.958	−2.10	−0.909
HB3	50%	1.312	1.872	1.152	1.026	1.330	0.884	−2.10	−0.880
	0%	1.157	1.801	1.040	1.171	1.178	0.797	−2.10	−0.815
	100%	1.254	1.910	1.033	1.134	1.581	1.033	−2.10	−1.022
HB1	50%	1.178	1.910	0.894	1.210	1.338	0.894	−2.10	−0.896
	0%	1.118	1.910	0.789	1.270	1.113	0.749	−2.10	−0.759

and (2) stiffness factors and fixed-end moments for haunched beams having T-sections based upon the formulation presented in Tena-Colunga (1996) might be used with confidence for the analysis of such beams. In fact, the design tables and correction curves presented in Tena-Colunga and Zaldo-García (1994) for haunched and tapered T-beams might be used with confidence, granted that there are no abrupt changes in the shape of the cross section (relative aspect ratio for the flange and web), as the stiffness coefficients for nonprismatic beams depend on the shape of the cross section, as demonstrated and shown before in Tena-Colunga (1996).

Finally, the discusser wants to state that, fortunately, there are some good analytical methods that can be used to study haunched beams, so he is not married with a single method. In some instances the finite-element method would be more appropriate to study haunched beams, in fact, in Tena-Colunga (1996) the discusser used 3D nonlinear finite elements to obtain load-deformation curves for steel frames with haunched beams using pushover analyses. In other instances, the Bernoulli–Euler beam theory presented by Tena-Colunga (1996), or any other suitable method already presented in the literature (some of them briefly discussed in Tena-Colunga (1996), a new one presented in the paper discussed here) would constitute a better option.

Appendix: Cross-Section Functions Used to Model Linearly Tapered T-Beams

The notation used in the following equations is in agreement with the one used Tena-Colunga (1996):

$$\alpha=\frac{h_{w2}}{h_{w1}}-1 \tag{4}$$

$$h(z)=h_{w1}\left(1+\alpha\frac{z}{L}\right) \tag{5}$$

$$A(z)=A_1+b_w h_{w1}\left(\alpha\frac{z}{L}\right) \tag{6}$$

$$A_{cy}(z)=A_{cy1}+b_w h_{w1}\left(\alpha\frac{z}{L}\right) \tag{7}$$

$$y(z)=\frac{b_w h(z)^2+b_f t_f(t_f+2h(z))}{2A(z)} \tag{8}$$

$$I_x(z)=\frac{1}{12}[b_w h(z)^3+b_f t_f^3]+b_w h(z)\left[\frac{h(z)}{2}-y(z)\right]^2+b_f t_f\left[h(z)+\frac{t_f}{2}-y(z)\right]^2 \tag{9}$$

References

Balkaya, C., and Citipitiouglu, E. (1997). "Discussion of 'Stiffness formulation for nonprismatic beam elements,' by A. Tena-Colunga." *J. Struct. Eng.*, 123(12), 1694–1695.

Prakash, V., Powell, G. H., and Filippou, F. C. (1992). "DRAIN-2DX: Base program user guide," *Rep. No. UCB/SEMM-92/29*, Dept. of Civil Engineering, Univ. of California at Berkeley, Berkeley, Calif.

Tena-Colunga, A. (1994). "Concerns regarding the seismic design of reinforced concrete haunched beams," *ACI Struct. J.*, 90(3), 287–293.

Tena-Colunga, A., and Zaldo-García, A. (1994). "Ductilidad de Marcos con Trabes Acarteladas y Columnas de Sección Variable." *Rep. No. FJBS CIS-94/04*, Centro de Investigación Sísmica, AC, Fundación Javier Barros Sierra, Mexico City (in Spanish).

Tena-Colunga, A. (1996), "Eficiencia de marcos con elementos de sección variable ante cargas laterales." *Proc. X-Congreso Nacional de Ingeniería Estructural*, Mérida, Mexico, Vol. I, pp. 183–193, November (in Spanish).

Closure to "Behavior and Modeling of Nonprismatic Members Having T-Sections" by Can Balkaya

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The writer would like to thank the discusser for his comments and comparison of the results on the paper. The discusser states that the variation of the centroidal axis within the length was taken into account in the formulation but these equations were not previously included in Tena-Colunga (1996) for space constraints and the given reference is in Spanish. The discusser says that the arching action is not completely ignored in such modeling in his study. Comparing the results with the three-dimensional finite-element model (3DFEM) and his haunched beam, Bernoulli–Euler (HBBE) model or fixed-end moments in Fig. 2, there are large differences with the 3DFEM results as the haunched depth increases. The discusser states that "the arching action becomes important as the haunched depth increases," and both methods roughly agree for depth ratios $R \leqslant 0.6$ for fixed-end moments for case 15. Therefore, $R > 0.6$ the arching effect becomes important and needs to be considered in the modeling. However, the formulation by discusser only considers the arching effect as the variation of the centroidal axis within the length and not completely considered in the formulation.

Tena-Colunga states that there are available design tables that are published to compare the 3DFEM results. However, as seen from Figs. 1, 2, 3, 4, and 5, there is a big difference. Therefore, considering full arching action in 3D models is important. Most importantly, there is coupling between axial force and moments. However, Tena-Colunga formulation considers that the bending, axial, and torsional flexibility terms are uncoupled.

Thus, for a design engineer, it is better to consider the 3DFEM results including completely arching action depending on haunched depth rather than available tables (except for small haunches) or to use the practical model proposed by Balkaya (2001) (2DPM) beam model within 15% deviation with 3DFEM results for a range of haunched depth $r=0.4$, 0.6, 1.0, 1.5, and 2.0 while haunched length α is changed from 0.1, 0.2, 0.3, and 0.4 for T beams having symmetrical haunches.

The results of these studies show that arching action is important, especially for deep haunches. A significant axial thrust is produced as a result of arching action. Most importantly, there is a strong coupling between bending moment, shear, and axial forces while this coupling is not included in the Tena-Colunga formulation.

3DFEM studies by Balkaya (2001) include the following:

1. consider full arching action;
2. local stress concentration (as the change of neutral axis);

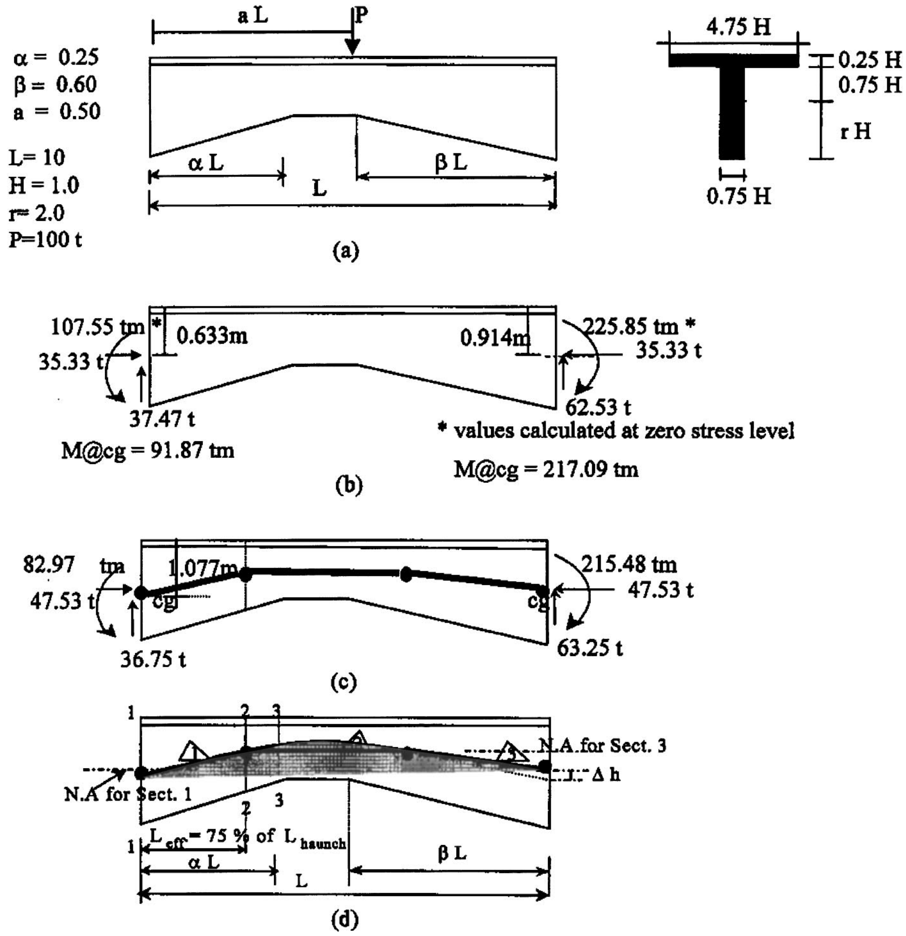

Fig. 1. Results in unsymmetrical haunch beam: (a) geometry and loading; (b) FE model; (c) practical model; and (d) proposed model with modification

3. consider the axial forces produced by the result of arching action;
4. consider the coupling effect between axial force, shear force and moments;
5. consider all cross-sectional effects;
6. variation of neutral axis; and

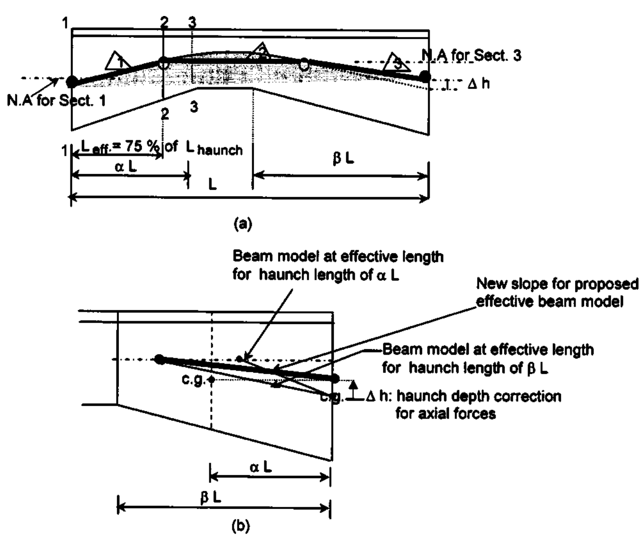

Fig. 2. Modeling of unsymmetrical haunch beam

7. consider nonlinear stress distribution especially beam–column connection for a frame analysis as well as in the calculation of zero stress point location.

On the other hand, HBBE studies by Tena-Colunga include the following:

1. consider cross-sectional effects as the variation of moment of inertia, and b/h ratio as variation of neutral axis;
2. do not consider the stress concentrations;
3. do not consider the coupling between axial force, shear force, and moments;
4. do not consider the true location of zero stress point at the fixed ends;
5. variation of neutral axes is considered only depending geometry, and does not depend on stress distribution; and
6. zero stress point location at the ends is important for the coupling between axial force and moments that is not considered in the true location considering nonlinear stresses. For this reason in the proposed model (2DPM) by Balkaya (2001) a correction for haunched depth is applied especially affecting the values of axial thrust depending on haunched depth.

For unsymmetrical arch formation with the help of Fig. 11(b) Tena-Colunga states that author decided not to compare his study with the results presented in Tena-Colunga (1996) for whatever reason. The reason is that part was taken out of the final copy before publishing to shorten the paper because of many figures. That part is added as accepted by the editor before the shortening process and is as follows.

Nonprismatic Members with Unsymmetrical Haunches

As a general case, beams having unsymmetrical haunches (Fig. 1) and loaded with a concentrated force are also investigated. These model studies are different than the models studied before due to the unsymmetrical haunches at the ends. To take care of this, as found from the finite-element analyses, the zero stress points are located at different levels at both ends. The main reason is that in unsymmetrical haunches, in addition to nonlinear stress distributions, the formation of the haunch is similar to a unsymmetrical arch depending on haunched depth and lengths, and having different haunch stiffnesses at the ends. In the case of unsymmetrical haunches, zero stress levels are different deviations at both ends and axial forces act with an eccentricity although they are equal at each end due to the equilibrium requirement. For these reasons, the haunched depth is modified.

As a modification parameter, as shown in Fig. 11(b) it is suggested that Δh be applied at one end to make an unsymmetrical arch. Thus, one side is moved up at the longest haunch length end in the proposed model. Δh is calculated by considering the longer haunch part as a reference. It is the difference between the centroid values at the end sections corresponding to small haunch length within the longest haunched length as shown in the Fig. 11(b).

In the part ''How to Model a Haunch Part'' an addition as follows can be made.

Model with Unsymmetrical Haunches

The only difference from the symmetrical haunch model is that the end node location of the long haunch beam is increased with a height of Δh. This mainly affects only axial force values. The modeling of unsymmetrical haunches is shown in Fig. 2. This modification in arch depth is due to formation of an unsymmetrical arch as explained in Fig. 2.

Example with Unsymmetrical Haunches

A beam having unsymmetrical haunches with T-sections as shown in Fig. 11(a) is subjected to a point load at the center. This model was used in the Tena-Colunga (1996) studies. The unsymmetrical haunched beam is analyzed by using 3D finite elements and the practical modeling technique as proposed by Balkaya (2001) in this study. In finite-element modeling the resultant forces are calculated at the zero stress levels at the fixed ends by using nodal force values, then transferred to T-section centroids for design and comparison of results with practical modeling. The results are shown in Fig. 1. There is a 10% difference in the end moment and shear values [Figs. 1(b) and 1(c)]. To make a modification for axial forces and to consider the formation of unsymmetrical arch in the model [Fig. 1(d)], the longer haunch section in the practical model is moved 0.359 units up, whereby the axial force value change from 47.53 to 33.61 and compared with 35.33 (5% deviation) in the finite-element model while end moment and shear values are within 10%.

In the formulation by Tena-Colunga, he gave the coefficients, for the 3D problem (T-haunched beam) by using Eqs. (4), (5), (6), (7), (8), (9), and (10) or by using Fig. 5 (Tena-Colunga 1996). Then by entering these coefficients in a computer program it is possible to obtain the end force and moments for one element. Even for this example or frames, it is not practical to use these coefficients. Furthermore, because of $r=2.0$ at both ends of the unsymmetrical haunch there will be large difference because of deep haunches. On the other hand, the proposed model, 2DPM (Balkaya 2001) for T-beams having symmetrical and unsymmetrical haunches will be practical and the results will be within 15% deviation with the results of 3DFEM.

Conclusion

Consider the small and deep range of T-section haunched beams arching action, stress distribution along the neutral axes, stress concentrations, nonlinear stress distribution, strong coupling between axial force, shear force and moments, zero stress location at the ends depending on haunch depths, and calculation of axial forces produced as a result of arching action, cross-sectional effects are observed that affect the behavior of T-haunched beams. Therefore, since the results are very different with the results of Tena-Colunga, the writer recommends using the 3DFEM results considering completely coupling effect as well as stress distributions rather than using tables by PCA or Tena-Colunga with large deviations, as shown in his discussion.

References

Balkaya (2001).

Tena-Colunga, A. (1996). ''Stiffness formulation for nonprismatic beam elements.'' *J. Struct. Eng.*, 122(12), 1484–1489.

Discussion of "Base-Isolated FCC Building: Impact Response in Northridge Earthquake" by Satish Nagarajaiah and Xiaohong Sun

September 2001, Vol. 127, No. 9, pp. 1063–1075.

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The purpose of this discussion is to point out an earlier study by Maison and Ventura (1992), who did a similar type of analysis and had the same conclusions as presented in the paper by Nagarajaiah and Sun (2001).

In the earlier study, correlative response analyses were conducted on a different CSMIP instrumented building that included consideration of hypothetical pounding at the isolator level. The pounding-modeling approach (Maison and Kasai 1990) used was the same as that by the writers. It is gratifying to see that the writers essentially confirmed the conclusions of the earlier work, namely:

1. Pounding at seismic gaps can produce peak story drifts, shears, accelerations, and floor spectra that are significantly greater than those from the no-pounding condition.
2. Pounding even has the potential to negate the benefits of base isolation, i.e., removing the undesirable effects of high-frequency motions.

Aside from the building analysis, the recorded motions at the FCC building exhibit a striking quality that the writers make only passing reference about: the remarkable difference in the NS and EW response spectra as generated from the recorded accelerations below the isolator level [Fig. 2(b)]. The associated EW acceleration response spectrum is much stronger than the corresponding NS spectrum. Such strong directivity for a site over 30 km from the Northridge epicenter is surprising. A study by Paret (2000) indicates that the apparent directivity effect was to have more intense NS shaking than EW shaking based on observed building damage (albeit for buildings closer to the epicentral region). The discussers would appreciate the writers discussion on this discrepancy. It would be also of interest if the writers show a plot of the free field ground motions response spectra and discuss if similar behavior is apparent there, also.

References

Maison, B. F., and Kasai, K. (1990). ''Analysis for type of structural pounding.'' *J. Struct. Eng.*, 116(4), 957–977.

Maison, B. F., and Ventura, C. E. (1992). ''Seismic analysis of base-isolated San Bernardino County building.'' *Earthquake Spectra*, 8(4), 605–633.

Nagarajaiah, S., and Sun, X. (2001). ''Base-isolated FCC building: Impact response in Northridge earthquake.'' *J. Struct. Eng.*, 127(9), 1063–1075.

Paret, T. F. (2000). ''The W1 issue.: Extent of weld fracturing during Northridge earthquake.'' *J. Struct. Eng.*, 126(1), 10–18.

Closure to "Base-Isolated FCC Building: Impact Response in Northridge Earthquake" by Satish Nagarajaiah and Xiaohong Sun

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The writers are thankful to the discussers for their interest in the paper, and for the comments. The writers regret the omission of the reference on the study by the discussers (Maison and Ventura 1992), which is a correlative response analysis of base isolated Foothill Communities Law and Justice Center (FCLJC) building based on the modeling approach for fixed base structures, formulation, and computer program developed by Maison and Kasai (1990) that the writers have referred to.

As the title suggests the main objective of the study by the writers was to evaluate the seismic response of base-isolated Fire Command and Control (FCC) building in Los Angeles, during the 1994 Northridge earthquake, and to model the three-dimensional behavior with nonproportional damping, one-sided eccentric impact with energy dissipation, and time-dependent gap [see Eqs. (1)–(7) and (13)–(16)]. The two- and three-dimensional formulation and the solution procedures for base-isolated structures, with nonlinearities and sudden changes in stiffness due to impact and with time-dependent gap, developed by the writers are new and accurate; the system identification, and the impact response features presented by the writers are new (see Figs. 4–9). The response due to impact is a nonlinear—also, time dependent in the study by the writers—phenomenon; hence, careful verification of the computed response is necessary. The computed responses, in the writers' study, are verified using the observed or recorded impact response of base isolated FCC in 1994 Northridge earthquake.

The discussers studied the response of base-isolated FCLJC building, in which no pounding has been observed or recorded to date, using the approach by Maison and Kasai (1990). The three-dimensional formulation and solution procedure developed by Maison and Kasai (1990) is for fixed base structures (used for modeling base-isolated structures by the discussers) with proportional damping, constant gap, and does not consider energy dissipation due to impact. Whereas, the method developed by the writers is for base-isolated structures with nonproportional damping, time dependent gap, and with due consideration given to energy dissipation due to impact [see Eqs. (1)–(7) and (13)–(16)]. Additionally, the discussers model double-sided pounding, whereas, the writers model one-sided impact. The hypothetical case of pounding studied by the discussers and the estimated response is unverified—as there is no recorded behavior of the base isolated FCLJC building with pounding to date.

The conclusions of the study by the writers shed light on the seismic response of FCC building during the Northridge earthquake: (1) revealed the fact that in the EW direction the impact ceased after certain duration and the entry bridge allowed free motion of the FCC building with behavior as a typical base-isolated building; and (2) there was no impact in the NS direction. The specific conclusion that the effectiveness of base isolation is reduced because of impact is similar to the conclusion of the discussers study. Nevertheless, this was neither the main objective nor the main conclusion of the study by the writers.

The claim by the discussers that the modeling, analysis, and the conclusions are essentially the same is inaccurate; since, it overlooks important differences, details, new developments by the writers for nonlinear time-dependent dynamic analysis of base-isolated structures with impact, and presentation of response features.

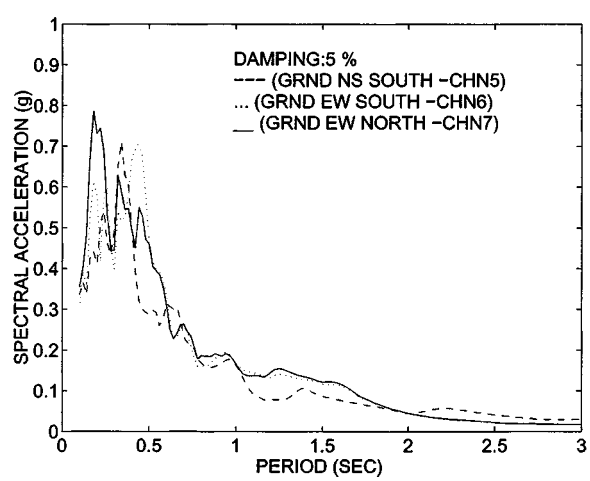

Fig. 1. Response spectra of the ground motion in the NS and EW direction

The significant difference between the NS and EW response spectra, shown in Fig. 2(b), is in error. The corrected response spectra are shown in Fig. 1. The difference between the NS and EW is nominal due to the absence of the directivity effect; it is worth noting that similar peak spectral acceleration of ~0.25 g result, in the case without impact, in the EW direction—as shown in Fig. 2(d), and in the NS direction—as shown in Fig. 3(b).

References

Maison, B. F., and Kasai, K. (1990). "Analysis for a type of structural pounding." *J. Struct. Eng.*, 116(4), 957–977.

Maison, B. F., and Ventura, C. E. (1992). "Seismic analysis of base isolated San Bernardino County Building." *Earthquake Spectra*, 8(4), 605–633.